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Essays on Financial Frictions

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Abstract

For the past forty years, the global economy has faced many challenges, including the increasing complexity and sophistication of financial markets, which created a new type of frictions that were in the origin of several recessive episodes such as the 2008 financial crisis. Such events were largely unexpected and much deeper than most theoretical macroeconomic models have predicted until that point, and the subsequent recovery was much slower. As a result, many macroeconomic models were heavily criticized for omitting key financial mechanisms and shocks stemming from the financial sector.

To overcome this gap, a large number of studies that started characterizing the cyclical properties of financial variables such as the risk premium, firms' equity and debt flows at an aggregate level has proliferated exponentially during the last decades, as well as the diversity of different aspects of the financial sector that were conveniently modelized, such as the impact of credit supply conditions and borrowing constraints. Among that extensive plethora of macrofinancial models, emerged a cluster of authors devoted to analyze the role of financial frictions as a source of business cycle fluctuations. This thesis contributes to that branch of the literature in three different ways.

Chapter 1 proposes a measure the welfare effects of financial shocks as a percentage of consumption in a real business cycle model and compare it with the welfare costs of a productivity shock. In this model firms use both equity and debt financing, and financial shocks are modelled as stochastic innovations in the probability to recover from the debt' liquidation value in case of default, as a result of a tightening of firms' financing conditions. The main results show that financial shocks explain a significant extent of the macroeconomic dynamics of real and financial variables observed during financial crises, including the events occurred during the recent 2008 financial recession. The findings also suggest that although the welfare costs imposed by financial shocks computed in this framework are small, their relevance as a source of disturbances in business cycles cannot be underestimated, since it is estimated that the welfare cost of a financial shock is approximately 4 times larger than the welfare cost of a standard productivity shock in the economy, assuming that there is a tax benefit over the gross interest rate and

that there is an equity payout cost that represents the rigidities affecting the substitution between debt and equity.

Chapter 2 establishes a comparison between the empirical time series of major macroeconomic aggregates during the period 1984Q1-2014Q2 with the simulated series computed from the Jermann and Quadrini (2012) model in terms of variability, persistence and amplitude of a productivity and a financial shocks, and between the respective impulse responses to each shock. The main goal is to infer if this model provides a solid theoretical framework capable of replicate and anticipate large and deep recessions caused by financial shocks, such as the 2008 financial crisis and subsequent Great Recession. I conclude that, despite being able to capture reasonably well the timing of the shocks and the consequent downfall of the real side of the economy that follows after the shock hits, the model is unable to replicate the magnitude, persistence and volatility found in the data. This paper also includes a brief survey which describes the sequence of events that led to the crash of the financial system in the U.S. in 2008 and characterizes the empirical behavior of financial and real variables during and after that period.

Chapter 3 applies a New Keynesian DSGE model in which the nominal interest rate is determined according to a truncated Taylor rule and includes eight different shocks based on Jermann and Quadrini (2012) and Smets and Wouters (2007) to study the role of an interest rate subsidy as a fiscal instrument able to circumvent completely (or at least partially) the effects of a binding zero lower bound in the economy. This paper shows that, generally, the standard structural shocks such as preference and technological shocks are large enough to activate the zero lower bound constraint, as well as financial innovations. However, these findings also suggest that, during and after recession periods, when the zero lower bound binds, those shocks produce wider responses of output and the major macroeconomic aggregates than they would produce in an unconstrained economy, in terms of persistence and volatility. This paper also shows that by manipulating an interest rate tax subsidy to overcome the zero bound problem, it is possible to circumvent the zero lower bound completely, but at the cost of not achieving the first best allocation. I also quantify the tax benefit that would be sufficient to partially neutralize

the effects of the zero lower bound constraint when different types of shocks affect the economy.

Keywords: Financial Frictions; Financial Shocks; Welfare Costs; Zero Lower Bound

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Contents

Introduction

For the past forty years, the global economy has faced many challenges, including the increasing complexity and sophistication of financial markets, that changed the pre-existent dynamics between the real economy and financial institutions and triggered the emergence of a new type of frictions that were in the origin of several recessive episodes such as the 2008 financial crisis. Such events were largely unexpected and much deeper than most theoretical macroeconomic models have predicted until that point, and the subsequent recovery was much slower. This failure in predicting the extension and depth of those recessions and neglecting the importance of the financial sector of a potential source of negative shocks had widespread implications for economic policy and economic performance, and many macroeconomic models were heavily criticized for omitting key financial mechanisms and shocks stemming from the financial sector.

To account for this gap in the literature, a lot of effort have been devoted to incorporate the financial sector into well established theoretical frameworks, such as Real Business Cycle (RBC) models or in New Keynesian Dynamic Stochastic General Equilibrium (DSGE) models. In these last decades, the number of studies that started documenting and characterizing the cyclical properties of financial variables such as the risk premium, firms' equity and debt flows at an aggregate level has proliferated exponentially, as well as the diversity of different aspects of the financial sector that were heavily scrutinized and discussed, such as the impact of credit supply conditions, risk aversion quantification, economic agents' expectations about future policy actions and the effect of unconventional interventionist policies from the central banks such as quantitative easing (QE) policies targeting yield curve or risk spreads. Among that extensive plethora of macrofinancial models, emerged a cluster of authors devoted to analyze the role of financial frictions as a source of business cycle fluctuations. This thesis contributes to that branch of the literature in three different ways.

Chapter 1 was mainly motivated by the necessity to quantify the welfare losses that households must face when are confronted with financial frictions and rigidities that affect firms' ability to borrow from financial intermediaries. Despite the fact that it is originated in the financial sector, that enforcement constraint imposed to firms and subject to random disturbances compromises the firms' flexibility to change their financial structure, i.e. decide the composition of debt and equity. Since those disturbances that limit the firms' capacity to borrow, they are referred to as "financial shocks". The major goal in this Chapter is to measure the welfare effects of financial shocks in a model with pecking order at the firm level, based on the model of Jermann and Quadrini (2012), and present these results in relative consumption units.

Although the results obtained are significantly small, they are consistent with other important references related to the welfare analysis, such as Lucas (1987), for example. According to my results, an agent in an economy affected only by the financial shock would have to increase his consumption in 0.0245 per cent in order to be as well off as in a frictionless economy without any type of shock, assuming that the interest rate benefit and the equity payout cost are active in this economy. In comparison, the welfare cost imposed by the isolated productivity shock is estimated to represent a sacrifice of approximately 0.0013 per cent of the consumption of an agent of that economy in comparison to the frictionless economy, which is almost 19 times smaller than the welfare cost imposed by the financial cost alone.

The importance of financial shocks as a source of serious economic downturns is nowadays widely accepted by several authors, that generally agree that the vast bulk of movements in aggregate real economic activity during the recent depressive episodes such as the Great Recession were due to financial frictions.

Although the literature related with financial and banking shocks has expanded significantly and the number of theoretical models that aim to explain how those financial frictions affect the real side of the economy has proliferated at a steady pace since the 2008 financial crisis, not all of those frameworks are equally efficient at replicating the behavior of financial and real variables during those episodes, especially in terms of volatility and magnitude of the shocks.

Therefore, the main goal of Chapter 2 is to use the RBC model proposed by Jermann and Quadrini (2012) to compare the empirical time series of major macroeconomic (financial and real) aggregates against the simulated series obtained using that model, in terms of persistence, propagation and amplitude effects caused by a financial and a productivity shock. The impulse response functions are also computed in relation to both shocks. The ultimate goal is to infer if this framework is able to replicate the behavior of those financial and real variables during and after events as the 2008 financial crisis as it is described in the data.

Since the Great Recession originally started in the U.S. with the subprime financial crisis, I focus essentially on U.S. data. Although the full sample ranges between 1952.I-2014.II, I focus my analysis essentially on the period between 1984.I - 2014.II, because it includes the time range originally chosen by Jermann and Quadrini (2012) (1984.I - 2010.II) to calibrate their model and perform simulations. One of the major exercises of this paper is precisely to quantify and evaluate the main changes in the calibration of the model caused by the inclusion of this new set of observations in the original sample, and also study the performance of the model while replicating the data, specially in terms of the response of the major aggregates to exogenous shocks.

In order to characterize the general environment that surrounded the 2007-2009 financial crisis, a short survey is also included in this paper, in which I describe the main causes, triggers and consequences of that recession, since its early beginnings with the burst of the housing bubble at the end of 2006, including the start of the collapse of the financial system at the end of 2007 with the subprime crisis, and until the later period of worldwide economic depression which is now denominated the Great Recession.

I conclude that overall, the Jermann and Quadrini (2012) model is able to explain the empirical evidence associated with the time sample considered (1984.I-2010.II). The model seems to replicate relatively well the empirical evidence on volatility and on correlation between the simulated variables and data. However, after updating the data until 2014.II and after recalibrating the model the model seems unable to reproduce more deep, prolonged and deep recessions triggered by productivity or financial shocks such as the 2008 financial crisis.

The motivation for Chapter 3 stems from the fact that, since the Great Recession, the major economies such as the U.S. have experienced an unusual combination of features that include the weak growth of the GDP, low core inflation, and a real interest rate close to zero. The process of recovery has been much slower than usual in most high-income countries and has lasted longer than in previous financial crises, leading to central banks' efforts to stabilize the financial system through less conventional quantitative easing measures, such as forcing the reference interest rates near zero by the end of 2008, and held them there for an undefined amount of time, allowing commercial banks to accumulate substantial cash reserves in their accounts. This pattern of excess liquidity in the market accompanied by a real interest rate close to zero implies the presence of a liquidity trap, under which conventional central bank open-market operations are ineffective as an expansionary policy instrument. Monetary authorities, therefore, began to consider alternative policy approaches for escaping the trap, including raising the inflation target, depreciating the currency and targeting long-term interest rates directly. Less conventional policies were also considered, such as an excess reserve tax for commercial banks.

Chapter 3 studies the role of a tax benefit on the interest rate as an instrument to overcome a liquidity trap. It also considers whether the imposition of a zero lower bound constraint amplifies or not the negative impact of a financial shock in the economy, particularly during and after the 2008 financial crisis. To accomplish this goal, the structural DSGE model developed by Jermann and Quadrini (2012) is used, originally estimated with Bayesian maximum likelihood techniques. This structural approach follows closely the model estimated by Smets and Wouters (2007), but with the addition of financial frictions and financial shocks. Then, I compare the main findings against the results obtained in the most recent related literature, particularly the model estimated by Lindé et al. (2016). These authors update the Smets and Wouters (2007) by adding a financial accelerator mechanism, shocks originated in the financial sector and by introducing explicitly the zero lower bound into the model. They extend the model with a financial accelerator and allow for time-variation in the endogenous propagation of financial shocks. However, their approach to the financial shock and to the financial friction differ significantly from the approach presented by Jermann and Quadrini (2012), and they do not focus on the role of the interest rate subsidy as an attainable instrument to circumvent the effects of the zero lower bound, which is the fundamental premise of this paper.

In general, the findings in this Chapter suggest that, in an economy constrained by the binding presence of a zero lower bound imposed in a truncated Taylor rule by the monetary authority, the presence of an interest rate subsidy and other frictions (such as the equity payout cost, price and wages rigidities and investment adjustment costs) helps to amplify and propagate the effects of shocks of very different sources that can affect the economy.

Chapter 3 also shows that, generally, the standard structural shocks such as preference and technological shocks are large enough to activate the zero lower bound constraint, as well as financial innovations. However, these findings also suggest that, during and after recession periods, when the zero lower bound binds, those shocks produce wider responses of output and the major macroeconomic aggregates than they would produce in an unconstrained economy, in terms of persistence and volatility. These findings also show that by manipulating an interest rate tax subsidy to overcome the zero bound problem, it is possible to circumvent the zero lower bound completely, but at the cost of not achieving the first best allocation.

Chapter 1

The Welfare Effects of Financial Crises

1.1 Introduction

It is well documented and widely discussed that financial shocks played a crucial role in the recent 2007-2009 financial crisis. In fact, several authors such as Jermann and Quadrini (2012), Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Bernanke et al. (1999), Mendoza and Smith (2006) and Mendoza (2010) have shown that the financial sector was one of the major sources of propagation and persistence of business cycle fluctuations during the last 20 years, especially during recent events starting with the subprime crisis in the summer of 2007. This financial crisis inevitably led to an economic depression in which unemployment has risen sharply, and consumption and investment dropped substantially, at least for the most developed economies. But ultimately to what extent, in percentage terms, did the average consumer lost in welfare due to these financial frictions?

The main objective of this paper is to measure the welfare effects of financial shocks in a model with pecking order at the firm level, based on the model of Jermann and Quadrini (2012) [1], and present these results in relative consumption units. At

[1] Which was based on Jermann and Quadrini (2009).

a first stage, I build on their basic framework, which replicates simultaneously real aggregate variables and aggregate flows of financing such as debt and equity. I follow their method of construction of time series for the financial shocks from the model's enforcement constraint, using empirical data for debt, capital and output.

This paper contributes to a growing literature on welfare analysis, whose main reference in the last decades is given by the work of Lucas (1987), later updated by Lucas (2003). Lucas computed an estimate of the welfare gain associated with the elimination of business cycles, and translated the comparison between an economy with and without cycles into a comparison between an estimated time series representation of the actual postwar U.S. consumption path and the trend part of the representation. In order to establish a welfare comparison, Lucas assumed an infinitely lived risk-averse consumer, endowed with a stochastic consumption stream, who maximizes expected utility. Lucas' calculation assumed also that aggregate risk is shared equally among all individuals.

Contrary to Lucas (1987), one of the main goals of this analysis is to measure the isolated welfare effects of the financial shocks, not just the welfare effects of business cycle fluctuations over the stream of consumption. That is a reason why I perform my welfare cost calculations using a Real Business Cycle (RBC) model that considers two exogenous shocks: a productivity and a financial shock. The productivity cost is included in order to set the standard RBC model as a benchmark, and in order to compare the direction and the magnitude of the financial shock against those of a shock more studied and established in the literature. Therefore, I estimate four different calibrated models to compute the welfare effects: a model with both the productivity and the financial shocks, a model with only the financial shock, a model with only the productivity shock (the real RBC benchmark economy, in this case), and a model without any shock. Through all the simulations of all types of models I compute the expected utility in the steady state, and then use those calculations to perform the welfare cost analysis, following a similar method as the one described by Lucas (1987).

In the Jermann and Quadrini (2012) framework the introduction of an interest rate tax benefit (in order to accommodate the existence of the U.S. debt tax shield that makes all the interest paid on corporate debt tax deductible to the firms) and the introduction of a quadratic adjustment cost function on equity payout are crucial to the amplification and propagation effects of the financial shock in the simulated economy, largely because they affect the way firms react to both shocks. The link between the financial shock and the behavior of real variables, especially labor, depends largely in the parameters associated with these frictions. Also, these frictions determine the firm's optimal level of debt even in the absence of stochastic shocks. Therefore, I perform the different simulations of the model assuming the presence or absence of those frictions. In total, assuming that both the financial and the productivity shocks are exogenous, I run sixteen simulations to cover all possible combinations of frictions and shocks that can change the households' steady state and lifetime welfare situation.

Since the introduction of the two financial frictions (the interest rate subsidy and the equity payout cost) imply a significant change in the steady state levels of the majority of the variables of the model, I applied a correction procedure in the calculation of the expected utilities of those sixteen simulations, in order to avoid the production of biased welfare effects estimates. I also include a more detailed characterization of the equilibrium in this framework in order to obtain a better understanding of the behavior of the model in the steady state when we consider different scenarios for the two financial frictions.

The results obtained are significantly small, and this fact is consistent with a large branch of the literature related to the welfare analysis, like the results obtained by Lucas (1987), for example. According to my results, an agent in an economy affected only by the financial shock would have to increase his consumption in 0.0245 per cent in order to be as well off as in a frictionless economy without any type of shock, assuming that the interest rate benefit and the equity payout cost are active in this economy. In comparison, the welfare cost imposed by the isolated productivity shock is estimated to represent a sacrifice of approximately 0.0013 per cent of the consumption of an agent of that economy in comparison to the frictionless economy, which is almost 19 times smaller than the welfare cost imposed by the financial cost alone.

The rest of the paper is structured as follows. Section 2 presents a brief literature review about the welfare analysis and the role of financial shocks over the economy. Section 3 describes the standard RBC model presented by Jermann and Quadrini (2012) with financial frictions and financial shocks. Section 4 defines and characterizes the equilibrium of the benchmark economy and studies different cases depending on the inclusion or exclusion of the two financial frictions. Section 5 describes the parameterization and estimation methods. Section 6 describes the welfare effects analysis, presenting the methodology used and the main findings. Section 7 concludes.

1.2 Literature

In the welfare effects' literature, one of the main references is given by Lucas (1987), who calculated an estimate of the welfare gain associated with the elimination of business cycles. He translated the comparison between an economy with and without cycles into a comparison between an estimated time series representation of the actual postwar U.S. consumption path and the trend part of the representation. In order to establish a welfare comparison, Lucas assumed an infinitely lived risk-averse consumer, endowed with a stochastic consumption stream, who maximizes expected utility. Lucas' calculation assumed also that aggregate risk is shared equally among all individuals. His estimates implied welfare gains of no more than a very small fraction of about one-twentieth of 1 per cent of consumption. With different specifications this value changes slightly, but not significantly, e.g, for logarithmic utility the welfare gains are approximately 0.008 per cent of consumption.

However, one of the main criticisms pointed to the Lucas method is the assumption regarding consumer homogeneity, both in terms of preferences, employment status and wages. Several authors tried to overcome that handicap, such as Krusell and Smith (1999), that also computed the welfare effects of eliminating business cycles, but assuming consumer heterogeneity which arises from uninsurable and idiosyncratic uncertainty in preferences and employment. Their calculations lead them to conclude that the steady state average gains from eliminating business cycles are higher than the Lucas (1987) representative-agent model estimates. Using a model with a two-state process both for employment and for the productivity shock, they obtained a long-run welfare gain from eliminating cycles of approximately 0.138 per cent, which is much higher than the welfare gain of 0.008 per cent of consumption estimated by Lucas (1987). However, in order to infer about the welfare effects of eliminating business cycles for different groups of agents, Krusell and Smith (1999) extended their calculations to include transition paths experiments, and they concluded that those welfare gains are very small and even negative for almost all groups covered in their estimations. In other words, on average, there is a welfare loss from eliminating cycles, according to their model.

One needs to consider that their framework has several weaknesses: the precise amount of those welfare effects depends largely on the calibration of the model, but also on the initial aggregate capital stock and aggregate exogenous state. Furthermore, their model treats both the aggregate and the idiosyncratic shocks as exogenous, which forces the construction of two different models to establish a comparison between an economy with cycles and one without, using different assumptions regarding production possibilities and idiosyncratic shocks that are quite controversial. They also do not consider idiosyncratic wage heterogeneity among agents, which can constitute a much larger force to stimulate business cycle fluctuations than aggregate wage risk, leading to larger estimates of the welfare costs of business cycles.

Other authors, such as Cooley and Hansen (1991), used a similar comparison approach to estimate the revenue consequences and the welfare costs caused by the inflation tax, introducing an equilibrium growth model with a cash-in-advance constraint. Through this model, they examined how the presence of other distorting taxes affects the welfare costs of inflation and the revenue that can be raised by the inflation tax. Their welfare methodology consisted in computing the percentage increase in consumption that an individual would require to be as well off as under the First Best allocation, which is one of the main principles also adopted in this paper. Although the results confirm the widely held view that a permanent reduction of the inflation rate to zero has positive welfare consequences, they also found that the presence of other distorting taxes approximately doubles the estimated welfare cost of the inflation tax and decreases the revenue potential of the tax, and that policies which aim to eliminate the inflation only temporarily actually make the economy worse off because of the substitution effect that occurs. However, their model differs substantially from mine due to the presence of money and due to the absence of uncertainty. They only compute welfare costs considering different situations for the economy in the steady state, and do not include a dynamic analysis of the transition between those different steady states.

There are authors such as Alvarez and Jermann (2000) that propose an alternative method to measure the cost of business cycles, by using an approach that does not require the specification of preferences and instead uses asset prices. The method is based on the marginal cost of consumption fluctuations, the per unit benefit of a marginal reduction in consumption fluctuations expressed as a percentage of consumption. They show that this measure is an upper bound for the benefit of reducing all consumption fluctuations. We also clarify the link between the cost of consumption uncertainty, the equity premium, and the slope of the real term structure. They find that consumers "would be willing to pay a very high price for a reduction in overall consumption uncertainty." However, for consumption fluctuations corresponding to business cycle frequencies, we estimate the marginal cost to be about 0.55% of lifetime consumption based on the period 1889-1997 and about 0.30% based on 1954-97.

In relation to the financial shocks' literature, i.e. the field of the literature that suggest that shocks originated in the financial sector can play an important role as a source of macroeconomic fluctuations, it is always important to mention the major contribution of the model of bank runs developed by Diamond and Dybvig (1983), since this is a model which helped to clarify to a great extent the behavior of banks, showing how these are able to create liquidity by investing in long-run projects and offering more liquid contracts.

However, since my model is most closely related to Jermann and Quadrini (2012), this model embodies characteristics and influences from the branch of the literature that defend that these shocks could constitute one of the major sources of macroeconomic fluctuations, such as Bernanke and Gertler (1989), Bernanke et al. (1999), Kiyotaki and Moore (1997), Kiyotaki and Moore (2012), Mendoza and Smith (2006), Mendoza (2010) and Christiano et al. (2010). In common, all these models assume that shocks that directly or indirectly affect the financial sector lead to the generation of business cycles, either due to the existence of credit and collateral constraints or other types of frictions that prevent several economic agents to access capital and credit markets, leading to heterogeneity in the access to those markets, both for firms and for households. These authors show that this type of shocks, based on the interaction between credit limits and asset prices is a powerful transmission mechanism by which the effects of those shocks persists, amplify, and spill over to other sectors of the economy. Although some of these models directly specify an intermediary sector, they do not consider shocks that directly affect the financial sector, as in Jermann and Quadrini (2012), but they rather consider productivity or technology shocks that triggers collateral constraints on domestic agents. Furthermore, the Jermann and Quadrini (2012) framework also differ from those models by assuming that firms are allowed to have negative equity payouts, which can be interpreted as new equity issues.

One of the assumptions that constitute a key determinant of the impact of financial frictions in this environment is the trade-off between debt and equity, formalized as a quadratic equity payout cost function. Modigliani and Miller (1958) in their classical model showed that, if capital markets are frictionless, the total value (equity plus debt) of a firm is independent of the firm's capital structure. However, in this environment, financial frictions are introduced in the economy, and therefore, rigidities between debt and equity must be taken into account, and are modelled as a cost to the firm. The acknowledgement of this trade-off in the capital structure of firms is well established in the corporate finance literature, as can be verified through the research of several authors, for example, Leary and Roberts (2005), Scott Jr. (1976) and Miller (1977).

It is also important to consider whether the omission or presence of other distorting taxes besides the interest rate subsidy considered in this framework is determinant for the impact of the financial shocks over the behavior of the real variables. Other authors such as Cooley and Hansen (1991), when studying the welfare costs of the inflation tax, examined how the presence of distorting taxes on labor and capital income affects those welfare costs and the revenue that can be raised by the inflation tax, using an equilibrium growth model with a cash-in-advance constraint. Their results show that the introduction of those taxes into their model approximately doubles the estimated welfare cost of the inflation tax and decreases the revenue potential of the tax. Cooley and Hansen (1991) also consider if the timing and duration of the shocks produce significant changes in the welfare costs estimates of inflation, and concluded that policies that reduce the inflation rate to zero only temporarily actually make the economy worse off because of the intertemporal substitution that takes place.

1.3 Benchmark Model

I use the framework presented by Jermann and Quadrini (2012), a standard RBC model with financial frictions and financial shocks. The benchmark model consists of a continuum of identical consumers and a continuum of identical firms. This section starts to specify the firm's problem and then the consumer's firm.

1.3.1 Firms

There is a continuum of firms, in the $[0, 1]$ interval. The production function is given by $F(z_t, k_t, n_t) = z_t k_t^{\theta} n_t^{1-\theta}$, where z_t is the stochastic level of productivity, k_t is the input of capital and n_t is the input of labor. It is assumed that z_t follows the stochastic process: $z_t = \rho_z z_{t-1} + \varepsilon_z$, where ε_z is assumed to be i.i.d., have a zero mean and variance equal to σ_z. Innovations to z_t are defined as the productivity shocks in this framework, and they affect all firms. The capital stock k_t is chosen at time $t-1$ and therefore predetermined at time t. Firms have more flexibility when choosing the input of labor n_t at time t.

The law of motion of capital is given by $k_{t+1} = (1-\delta)k_t + i_t$, where i_t is investment and δ is the depreciation rate.

It is assumed that firms use both equity d_t and debt b_t in order to finance themselves. In this model, one of the main features is the pecking order in the financial decision of firms between equity and debt. Debt is preferred to equity because of its tax advantage, but the firms' ability to borrow of limited by an enforcement constraint which is subject to random disturbances, designated as "financial shocks". Given that r_t is the net interest rate, the effective gross interest rate for the firm is $R_t = 1 + r_t(1-\tau)$, where τ denotes the tax benefit. This tax deduction enjoyed by firms is paid for by a lump-sum tax on households. The tax advantage also implies that the aggregate amount of debt B_t will always be strictly positive. According to Amdur (2008) this creates a wedge between the gains a firm receives from issuing a bond and the price that consumers pay for a bond. Since the firm perceives that it can issue bonds at a favourable price due to the tax benefit, it has an incentive to issue a positive amount of debt. However, the price of corporate debt increases to consumers with the amount issued because of the monitoring costs to ensure repayment supported by the investors/consumers, according to their assumptions. Therefore, as the price of debt increases, consumer demand for corporate bonds decline. The tradeoff between the tax advantage and monitoring costs is crucial to define the equilibrium level of debt and the interest rate r_t in equilibrium.

In order to finance working capital, in addition to the debt, b_t, firms can raise funds with an intra-period loan l_t. According to Jermann and Quadrini (2012), "working capital is required to cover the cash flow mismatch between the payments made at the beginning of the period and the realization of revenues" (after production). Therefore, according to Lucas timing, the intra-period loan is repaid at the end of the period and there is no interest, by assumption. The assumption of the existence of this intra-period loan is a short cut to the fact that firms carry "cash" or "liquidity" to the next period. The loan is then used to pay the equity holders (including dividends) and to finance working capital (wages and investment), implying that the payments of dividends comes from previous periods earnings. Firms also issue equity shares s_t to its shareholders at the market price p_t.

Firms start the period with intertemporal debt b_t. Before producing they choose labor, n_t, investment, $i_t = k_{t+1} - (1-\delta)k_t$ and equity payout, d_t, and they issue new intertemporal debt, b_{t+1}. Firms also contract an intra-period loan l_t, since the payments to workers, suppliers of investments, shareholders and bondholders are made before the realization of revenues:

$$l_t = w_t n_t + i_t + d_t + b_t - b_{t+1}/R_t \tag{1.1}$$

Therefore, the condition above can be interpreted as the equivalent to a cash-in-advance (CIA) constraint in the standard literature.

The firm's budget constraint is the following:

$$b_t + w_t n_t + k_{t+1} + d_t = (1-\delta)k_t + F(z_t, k_t, n_t) + b_{t+1}/R_t \tag{1.2}$$

Using the previous two conditions follows that the intra-period loan is equal to the firm's production function/revenues, i.e. $l_t = F(z_t, k_t, n_t)$. However, this is only true as long as there is the tax benefit [2] ($\tau > 0$).

Since firms can default on their obligations and divert some of its own resources (specifically, the amount of liquidity available at period t, which corresponds to the intra-period loan $l_t = F(z_t, k_t, n_t)$), the ability to borrow in both time horizons is constrained by the limited enforceability of debt contracts. According to Jermann and Quadrini (2012), "the decision to default arises after the realization of revenues

[2] I will show this later in the characterization of the equilibrium, in Proposition 5 of Section 1.4.1.2.

but before repaying the intra-period loan". At this stage the firm holds a total amount of liabilities given by the intra-period loan plus the new intertemporal debt, i.e. $l_t + b_{t+1}/R_t$. In case of default, the only asset available for liquidation is the physical capital k_{t+1}, since firms can easily divert the total amount of liquidity available at that period.

The renegotiation process between the firm and the lender in the event of default, in this economy, takes place without the introduction of a well defined system of financial intermediation. If the firm defaults, the lender acquires the right to liquidate the firm's capital. It is assumed that at the moment of contracting the loan the liquidation value of physical capital k_{t+1} is uncertain. With probability ξ_t the lender will be able to recover the whole value k_{t+1} but with probability $1-\xi_t$ the recovery value is zero. Neither the lender nor the firm are able to observe the liquidation value before the actual default. Therefore, these two cases are considered separately to determine the renegotiation outcome. In order to do so, it is assumed that the firm has all the bargaining power in the renegotiation and the lender gets only the threat value. The complete description of the renegotiation process is included in Section 1.7.1 of the Appendix. Therefore, the enforcement constraint faced by firms is given by:

$$\xi_t \left(k_{t+1} - \frac{b_{t+1}}{1+r_t} \right) \geq l_t$$

When this constraint is binding, higher debt, either intertemporal or intra-temporal, tightens the enforcement constraint, and a higher stock of capital has the opposite effect. When this enforcement constraint does not bind [3], that is, when $l_t < k_{t+1} - b_{t+1}/(1+r_t)$, it imposes a liquidity cost that acts in the economy as an inflation tax: if the disposable amount of liquidity $l_t = F(z_t, k_t, n_t)$ given by the production is not sufficient to make the payment that leaves the lender indifferent between liquidation and keeping the firm in operation, then that implies that the firm will not be able to finance working capital imposing an implicit tax on its workers, investors, shareholders and bondholders.

[3]This happens when $\tau = 0$, as I show in Proposition 6 of Section 1.4.1.2, in the characterization of the equilibrium.

The probability ξ_t is stochastic and follows the process $\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_\xi$. It is common for all firms and depends on market conditions. We can interpret ξ_t as the probability of non-default by the firms. Stochastic innovations to this variable will be considered the "financial shocks" of this model, since it affects the tightness of the enforcement constraint, therefore affecting the borrowing capacity of the firm. Hence, so far, we have two sources of aggregate uncertainty in this model: productivity shocks, z_t, and financial shocks, represented by ξ_t. Since there are no idiosyncratic shocks, I will focus on the symmetric equilibrium solved for a representative firm.

In order to better understand how ξ_t affects the financing and production decision of firms, the enforcement constraint can be rewritten. In order to do so, and to eliminate $k_{t+1} - b_{t+1}/(1+r_{t)}$ from the enforcement constraint, the budget constraint must also be written:

$$k_{t+1} - \frac{b_{t+1}}{1+r_t} = (1-\delta)k_t - b_t - w_t n_t - d_t + b_{t+1}\left(\frac{r_t \tau}{(1+r_t)R_t}\right) + F(z_t, k_t, n_t) \quad (1.3)$$

Substituting this condition into the enforcement constraint, we get the following condition:

$$\left(\frac{\xi_t}{1-\xi_t}\right)\left[(1-\delta)k_t - b_t - w_t n_t - d_t + \frac{b_{t+1}}{(1+r_t)}\left(\frac{r_t \tau}{R_t}\right)\right] \geq F(z_t, k_t, n_t) \qquad (1.4)$$

It is assumed that at the beginning of the period k_t and b_t are given, thus the only variables that are under the control of the firm are the input of labor, n_t, the equity payout, d_t and the new intertemporal debt, b_{t+1}.

Assuming as a starting point for the analysis a stationary state in which the enforcement constraint is binding and the firm plans to keep the production structure unchanged (especially labor), then from the condition above it is possible to conclude that a negative financial shock (lower ξ_t) implies either a reduction in equity payout

d_t or a decrease in the employment level n_t. The effect over the intertemporal debt for the next period b_{t+1} is more unclear, due to the positive term $\frac{b_{t+1}}{(1+r_t)}\left(\frac{r_t\tau}{R_t}\right)$, but adding that to the left hand term of the budget constraint, $\frac{b_{t+1}}{1+r_t}$, gives a negative response to a negative financial shock. In other words, in order to accommodate the effects of the negative financial shock in the level of production, the firm is forced to choose between increasing its equity or decreasing the new intertemporal debt. However, if for some reason the firm is not able to reduce d_t or b_{t+1}, due to the degree of rigidity with which the firm can change its financial structure, the only option left is to reduce the employment level n_t.

This analysis become more clear if we consider the case where $\tau = 0$. In that case, the above expression reduces to:

$$\left(\frac{\xi_t}{1-\xi_t}\right)\left[(1-\delta)k_t - b_t - w_t n_t - d_t\right] \geq F(z_t, k_t, n_t). \tag{1.5}$$

However, it is necessary to take into account that when the tax advantage is absent, the enforcement constraint may not bind in equilibrium. In that case, a negative financial shock may not necessarily imply a reduction in the equity payout or employment in order to keep the level of production of the economy unchanged.

On the other hand, in the case of a positive and persistent productivity shock the firm would like to both increase investment and rise its capital stock in the next period (to take advantage of higher expected productivity) and also would like to pay more to shareholders, to pass on the unexpected increase in lifetime profitability, i.e., to increase the equity payout. In order to fulfil these goals, and if debt is available and the cost of changing the equity payout cost is low or absent, the firm will borrow heavily instead of increase its equity to boost its capital stock and increase future payments to shareholders. If, however, the cost of adjusting equity is high, the firm will refrain from borrowing to avoid large volatilities in its payout. As a consequence, the firm's adjustment in equity payouts is slower, and the issuance of new debt will decrease both in magnitude and in frequency in time.

Therefore, whether the financial shock affects employment depends crucially on the trade-off between debt and equity when the firm has to decide the composition of its financial structure. In other words, the Modigliani-Miller theorem (see Modigliani and Miller (1958)) asserting the irrelevance of firms asset structures does not apply in this economy when the enforcement constraint binds.

To formalize the rigidities affecting the substitution between debt and equity, and in order to capture the frictions associated with paying dividends, I assume that the firm's payout is subject to a quadratic adjustment cost, following Jermann and Quadrini (2012) framework. Given that d_t is equity payout, the actual cost for the firm is given by:

$$\varphi(d_t) = d_t + \kappa(d_t - \overline{d})^2,$$

Where $\kappa \geq 0$, and $\overline{d}$ is a parameter equal to the long-run payout at the steady state.

According to Jermann and Quadrini (2012), the equity payout can be interpreted as a way of modelling the flexibility and "the speed with which firms can change the source of funds when financial conditions change, i.e. it can be interpreted as the financial cost in which firms have to incur due to the trade-off between equity and debt". In order to change the composition of its portfolio, the firm must incur in legal and accounting costs associated with issuing and repurchasing equity shares, as well as costs associated with adjusting their equity payouts to shareholders. The parameter κ is a measure of the scale of the adjustment cost of the firm's payouts to shareholders. The quadratic cost function implies that the adjustment cost is increasing in the deviation of today's equity payout from its long term steady-state payout target. The convex functional form of the adjustment cost given here is meant to mimic the empirical evidence that the preferences of managers turn to dividend smoothing over time.

The parameter κ is determinant to evaluate the impact of financial shocks over the rest of the economy. When $\kappa = 0$, the economy is almost equivalent to a frictionless economy, and in this case, "debt adjustments triggered by the enforcement constraint can be costlessly accommodated through changes in firm equity", according to Jermann and Quadrini (2012). When $\kappa > 0$, the substitution between debt and equity becomes costly and the adjustment to a different financial structure becomes slower, affecting the firm's production decisions. This is the main reason why financial shocks will produce non-negligible short-term effects on the production decision of firms.

Besides issuing non-contingent bonds b_t, firms also issue equity shares s_t, at the market price p_t. Let us denote the total amount of equity payout received from owning shares as d'_t (and the net equity payout simply as d_t), in order to solve the firm's optimization problem. Then, d'_t can be defined as $d'_t = s_t d_t + p_t(s_t - s_{t+1})$, as the sum of the total amount of dividends distributed by the shareholders in period t and the total net amount of share repurchases available in the same period.

The individual state variables are the capital stock, k, the debt, b, and the equity shares, s. The aggregate states are the equity payout d and the input of labor, n.

Given this, the firm's optimization problem is the following:

$$\max_{\{d_t, n_t, k_{t+1}, b_{t+1}, s_{t+1}\}_{j=0}^{\infty}} \left\{ E_t \left[\sum_{j=0}^{\infty} m_{t+j} d_{t+j} \right] \right\} \tag{1.6}$$

subject to:

$$(1-\delta)k_t + F(z_t, k_t, n_t) - w_t n_t + \frac{b_{t+1}}{R_t} + p_t s_t = b_t + \varphi(d'_t) + k_{t+1} + p_t s_{t+1}$$

$$\xi_t \left(k_{t+1} - \frac{b_{t+1}}{1+r_t} \right) \geq F(z_t, k_t, n_t)$$

Where m_{t+j} is the stochastic discount factor, which will be defined later in the hoseholds' optimization problem. This variable, along with the wage and the interest rate are all determined in the general equilibrium and are all taken as given by the representative firm.

Denoting by λ and μ the Lagrange multiplier associated with the budget and enforcement constraints, respectively, the Lagrangian of this problem is the following:

$$\mathcal{L} = E_t \left\{ \sum_{j=0}^{\infty} m_{t+j} \left[(s_{t+j}d_{t+j} + p_{t+j}(s_{t+j} - s_{t+1+j})) \right. \right. \tag{1.7}$$
$$+\lambda_{t+j}\left((1-\delta)k_{t+j}+F(z_{t+j},k_{t+j},n_{t+j})-w_{t+j}n_{t+j}+\frac{b_{t+1+j}}{R_{t+j}}+p_{t+j}s_{t+j}-b_{t+j}-\varphi(d'_{t+j})-k_{t+1+j}-p_{t+j}s_{t+1+j}\right)$$
$$\left.\left. + \mu_{t+j}\left(\xi_{t+j}\left(k_{t+1+j} - \frac{b_{t+1+j}}{1+r_{t+j}}\right) - F(z_{t+j},k_{t+j},n_{t+j})\right)\right]\right\}$$

Where $\varphi(d'_t) = d'_t + \kappa(d'_t - \overline{d})^2 = s_t d_t + p_t(s_t - s_{t+1}) + \kappa(s_t d_t + p_t(s_t - s_{t+1}) - \overline{d})^2$ is the equity payout cost function depending on the sum of the total amount of equity payout received from owning shares d'_t and $\varphi_d(d'_t) = s_t + 2\kappa s_t(d'_t - \bar{d}) = s_t + 2\kappa s_t(s_t d_t + p_t(s_t - s_{t+1}) - \bar{d})$ denotes the first derivative of that function in relation to d_t, the net equity payout.

The first-order conditions of this optimization problem for d_t, n_t, k_{t+1} and b_{t+1} and s_{t+1} are:

$$F_n(z_t,k_t,n_t) = w_t\left(\frac{1}{1-\mu_t\varphi_d(d'_t)}\right) \tag{1.8}$$

$$E_t m_{t+1}\left(\frac{\varphi_d(d'_t)}{\varphi_d(d'_{t+1})}\right)\left[1-\delta+\left(1-\mu_{t+1}\varphi_d(d'_{t+1})\right)F_k(z_{t+1},k_{t+1},n_{t+1})\right]+\xi_t\mu_t\varphi_d(d'_t)=1 \tag{1.9}$$

$$R_t E_t m_{t+1}\left(\frac{\varphi_d(d'_t)}{\varphi_d(d'_{t+1})}\right)+\xi_t\mu_t\varphi_d(d'_t)\left(\frac{R_t}{1+r_t}\right)=1 \tag{1.10}$$

$$E_t m_{t+1}\left(\frac{\varphi_d(d'_{t+1})}{\varphi_d(d'_t)}\right)[p_{t+1}-\varphi_s(d'_t)] = p_t \tag{1.11}$$

$$TVC: \lim_{t\to\infty} E_t\beta^t\lambda_t k_t = 0 \tag{1.12}$$

$$TVC: \lim_{t\to\infty} E_t\beta^t\lambda_t b_t = 0 \tag{1.13}$$

$$TVC: \lim_{t\to\infty} E_t\beta^t\lambda_t s_t = 0 \tag{1.14}$$

Substituting $\varphi_d(d'_t) = s_t + 2\kappa s_t(d'_t - \bar{d}) = s_t + 2\kappa s_t(s_t d_t + p_t(s_t - s_{t+1}) - \bar{d})$ back into each one of these conditions we obtain:

$$F_n(z_t, k_t, n_t) = w_t\left(\frac{1}{1 - \mu_t(1 + 2\kappa(d'_t - \bar{d}))}\right) \tag{1.15}$$

$$E_t m_{t+1}\left(\frac{1 + 2\kappa(d'_t - \bar{d})}{1 + 2\kappa(d'_{t+1} - \bar{d})}\right)\left[(1-\delta)+\left(1-\mu_{t+1}(1+2\kappa(d'_{t+1}-\bar{d}))\right)F_k(z_{t+1}, k_{t+1}, n_{t+1})\right]+\xi_t\mu_t(1+2\kappa(d'_t-\bar{d})) = 1 \tag{1.16}$$

$$R_t E_t m_{t+1}\left(\frac{1 + 2\kappa(d'_t - \bar{d})}{1 + 2\kappa(d'_{t+1} - \bar{d})}\right) + \xi_t\mu_t(1 + 2\kappa(d'_t - \bar{d}))\left(\frac{R_t}{1 + r_t}\right) = 1 \tag{1.17}$$

$$E_t m_{t+1}\left(\frac{1 + 2\kappa(d'_t - \bar{d})}{1 + 2\kappa(d'_{t+1} - \bar{d})}\right) p_{t+1} = p_t \tag{1.18}$$

First of all, it must be noted that none of these conditions depend directly on s_t or s_{t+1}, except through d'_t or d'_{t+1}. This fact will be used later when I show the irrelevance of the equity shares determination in the equilibrium.

According to Jermann and Quadrini (2012), each one of these conditions have an intuitive interpretation that provides further insights into the model. The first condition (1.15) determines optimality for labor, where the left hand side of the equation is the marginal productivity of labor and the right hand sight is the marginal cost of labor, as usual. However, in this case the marginal cost differs from the standard expression because the wage rate is augmented by a wedge that depends on the "effective" tightness of the enforcement constraint, given by $\mu\varphi_d(d)$. Due to this wedge, a tighter constraint increases the effective cost of labor and reduces its demand, decreasing employment. Therefore, this friction in labor demand constitutes the main channel of transmission of financial shocks to the real sector of the economy.

From the second condition (1.16) it is not immediately clear whether the determination of the optimal level for capital in this economy depends or not directly in the interest rate r_t. In this case, the marginal productivity of capital is also negatively influenced by the wedge $\mu_t\varphi_d(d_t)$, which implies that the tighter the enforcement constraint and the higher the cost of equity payout, the lower will be the stock of capital demanded in equilibrium. Solving the third condition (1.17) in order of $\xi_t\mu_t\varphi_d(d_t)$ and substituting the result into the second condition, we get an expression which resembles more closely the standard condition for the firm's demand for capital:

$$Em_{t+1}\left(\frac{\varphi_d(d_t)}{\varphi_d(d_{t+1})}\right)\left[(1-\mu_{t+1}\varphi_d(d_{t+1}))F_k(z_{t+1},k_{t+1},n_{t+1})-(r_t+\delta)\right]=\frac{R_t-(1+r_t)}{R_t} \tag{1.19}$$

From this condition shows that the determination of capital in equilibrium does not depend directly on ξ_t, and therefore its innovations (the financial shocks) do not have a direct impact over the stock of capital. However, the financial shocks have an indirect effect over the demand for capital through two channels: the multiplier μ_t and the interest rate R_t. The higher the μ_t, i.e., the tighter is the enforcement constraint, the higher the wedge over the marginal productivity of capital, and consequentially the smaller the demand for capital. If we consider the special case in which the cost of payout is zero, i.e., $\kappa = 0$, this implies that $\varphi_d(d_t) = \varphi_d(d_{t+1}) = 1$. Applying this condition, (1.9) and (1.19) become, respectively:

$$Em_{t+1}\left[(1-\mu_{t+1})F_k(z_{t+1},k_{t+1},n_{t+1})\right]+\xi_t\mu_t=1$$

$$Em_{t+1}\left[(1-\mu_{t+1})F_k(z_{t+1},k_{t+1},n_{t+1})-(r_t+\delta)\right]=\frac{R_t-(1+r_t)}{R_t}$$

As we can see from the second expression, even in the absence of the cost of equity payout, the demand for capital is always pinned down by the tightness of the enforcement constraint.

Condition (1.10) is the first order condition for debt, and it clarifies the relationship between the probability of repayment ξ_t and the enforcement constraint multiplier μ_t. Again, in order to better understand the intuition behind this condition, it is convenient to consider the special case in which the cost of payout is zero, i.e. $\kappa = 0$. In this case $\varphi_d(d_t) = \varphi_d(d_{t+1}) = 1$ and condition (1.10) becomes $R_t E m_{t+1} + \xi_t \mu_t R_t/(1+r_t) = 1$, which implies that the Lagrange multiplier μ_t is fully determined by the aggregate prices R_t, r_t and Em_{t+1}. It is also clear from conditions (1.8) and (1.9) that the production and investment choices of the firm (labor and capital choices, respectively) only depend on these prices. Taking as aggregate prices R_t, r_t and Em_{t+1} as given, condition (1.10) with $\kappa = 0$ implies that "there is a negative relationship between ξ_t and μ_t". This means that "lower liquidation values of the firm's capital make the enforcement constraint tighter". Then from condition (1.8) this negative relationship imposes that a higher μ_t implies a lower demand for labor. Changes in ξ_t only affect the investment policy of the firm given by condition (1.9) if they change the aggregate prices R_t and Em_{t+1}. But as long as the aggregate prices are not affected, the policy of the firm remains unchanged in these circumstances. It is also possible to observe that, in the absence of the equity payout cost, if the enforcement constraint is not binding in neither the current nor the next period, the Lagrange multiplier is $\mu_t = \mu_{t+1} = 0$. Then conditions (1.8) and (1.9) that determine the choice of labor and capital become $F_n(z_t, k_t, n_t) = w_t$ and $Em_{t+1}[1 - \delta + F_k(z_{t+1}, k_{t+1}, n_t)] = 1$, that is, the marginal productivities equal their marginal costs, just like in a standard real business cycle framework.

According to Jermann and Quadrini (2012) "this mechanism is reinforced when $k > 0$, since in that case it is costly to re-adjust the financial structure, and the change in ξ_t induces a larger movement in μ_t". In other words, a fall in the liquidation value of the firm's capital (a negative financial shock) will make the enforcement constraint tighter. In the first period t after the shock, this will lead to a reduction in the demand of labor. Then, starting from the next period $t + 1$, the input of capital will also decrease.

1.3.2 Households

The households sector is composed by a continuum of homogeneous households. According to Jermann and Quadrini (2012) assumptions, "households are the owners (shareholders) of firms. In addition to equity shares, they also hold non-contingent bonds issued by firms".

Households maximize their lifetime expected utility subject to their budget constraint. The complete representation of the problem is therefore given by

$$\max_{\{n_t, b_{t+1}, s_{t+1}\}_{j=0}^{\infty}} \left\{ E_t \left[\sum_{j=0}^{\infty} \beta^j U(c_{t+j}, n_{t+j}) \right] \right\} \tag{1.20}$$

subject to:

$$w_t n_t + b_t + s_t(d_t + p_t) = \frac{b_{t+1}}{1 + r_t} + s_{t+1} p_t + c_t + T_t$$

where where E_0 is the expectation operator conditional on the information set at time $t = 0$, Ω_0, c_t is consumption, β is the discount factor, and $T_t = B_{t+1}/[1 + r_t(1 - \tau)] - B_{t+1}/(1 + r_t)$ are lump-sum taxes financing the tax benefit of debt and firms.

After defining the Lagrangian of the problem, the first order conditions with respect to n_t, b_{t+1} and s_{t+1} are:

$$w_t U_c(c_t, n_t) + U_n(c_t, n_t) = 0 \tag{1.21}$$

$$U_c(c_t, n_t) - \beta(1 + r_t) E U_c(c_{t+1}, n_{t+1}) = 0 \tag{1.22}$$

$$U_c(c_t, n_t) p_t - \beta E(d_{t+1} + p_{t+1}) U_c(c_{t+1}, n_{t+1}) = 0 \tag{1.23}$$

$$TVC : \lim_{t \to \infty} E_t \beta^t \lambda_t b_t = 0 \tag{1.24}$$

The first condition determines the supply of labor and the second condition is the Euler equation that determines the interest rate. The third condition determines the price of shares, which states that, adjusted for dividends and discounting, the share price follows a first-order univariate Markov process and that no other variables can Granger cause the share price. Using forward substitution in this last condition I get the following condition:

$$p_t = E_t \sum_{j=1}^{\infty} \left(\frac{\beta^j . U_c(c_{t+j}, n_{t+j})}{U_c(c_t, n_t)} \right) d_{t+j}$$

According to Jermann and Quadrini (2012), this last equation confirms that firms' optimization is consistent with households' optimization. Therefore, "the stochastic discount factor is $m_{t+j} = \beta^j U_c(c_{t+j}, n_{t+j})/U_c(c_t, n_t)$".

1.4 Equilibrium

Now, a general equilibrium is specified in Definition 1.

DEFINITION 1 (Equilibrium): *Given that the aggregate states* $\boldsymbol{s}\dot{}\boldsymbol{t}$ *are the productivity* z_t*, the variable* ξ_t*, the aggregate capital* K_t*, and the aggregate bonds* B_t*, a competitive equilibrium is defined as an allocation* $\{c_t(\boldsymbol{s}_t), n_t(\boldsymbol{s}_t), k_t(\boldsymbol{s}_t)\}_{t=0}^{\infty}$ *all* $\boldsymbol{s}_t$*; debt and equity levels* $\{b_t(\boldsymbol{s}_t), d_t(\boldsymbol{s}_t)\}_{t=0}^{\infty}$ *all* $\boldsymbol{s}_t$*; and prices and policies* $\{w_t(\boldsymbol{s}_t), r_t, m_t(\boldsymbol{s}_t, \boldsymbol{s}_{t+1})(\boldsymbol{s}_t)\}_{t=0}^{\infty}$ *all* $\boldsymbol{s}_t$*, characterized by a set of functions for (i) households' policies* $c^h(\boldsymbol{s}_t)$*,* $n^h(\boldsymbol{s}_t)$ *and* $b^h(\boldsymbol{s}_t)$*; (ii) firms' policies* $d(\boldsymbol{s}_t; k_t, b_t)$*,* $n_t(\boldsymbol{s}_t; k_t, b_t)$ *and* $b_t(\boldsymbol{s}_t; k_t, b_t)$*; (iii) aggregate prices* $w_t(\boldsymbol{s}_t)$*,* $r_t(\boldsymbol{s})_t$ *and* $w_t(\boldsymbol{s}_t, \boldsymbol{s}_{t+1})$*; (iv) law of motion for the aggregate states* $\boldsymbol{s}_{t+1} = \Psi(\boldsymbol{s}_t)$*, that solve the problems of the firms and the households, such that all markets clear: (i) households' policies satisfy conditions* (21)*,* (22) *and* (23) *subject to the budget constraint given in* (20)*; (ii) firms' policies are optimal and satisfy conditions* (8)*,* (9)*,* (10) *and* (11) *maximizing the utility function subject to the firm's budget constraint and the enforcement constraint; (iii) the wage and the interest rates clear the labor and bond markets through firms' conditions* (8) *and* (10) *and* $m(\boldsymbol{s}_t, \boldsymbol{s}_{t+1}) = \beta U_c(c_{t+1}, n_{t+1})/U_c(c_t, n_t)$*; (iv) the law of motion* $\Psi(\boldsymbol{s}_t)$ *is consistent with individual decisions and the stochastic processes for* z_t *and* ξ_t*.*

1.4.1 Characterization of the Equilibrium

1.4.1.1 Equivalence Results on Allocations, Prices and Policies in Equilibrium

I now show that the set of implementable allocations when firms issue both equity shares and redistribute equity payout to its shareholders is the same as the set of implementable allocations when firms only issue the equity payout. In other words, I show that, for a firm, issuing equity shares is the same as issuing equity payout, by proving that every equilibrium allocations under $s_t \neq 1$ can be implemented under $s_t = 1$, using the policy instruments available, i.e. the equilibrium is neutral to the value of s, for all t. If there is an equilibrium set of allocations, prices and policies that can be decentralized for a certain sequence of $s_t \neq 1$, then the same equilibrium can be decentralized for a sequence of $s_t = 1$, for all t. But before show that result, it is necessary to define and characterize the set of implementable allocations, prices and policies when $s_t = 1$.

DEFINITION 2 (Set of implementable allocations) *Assuming that the equity shares are set as $s_t = s_{t+1} = 1$, the set of implementable allocations, prices and policies $\{d_t, n_t, c_t, k_{t+1}, b_{t+1}, w_t, r_t\}_{t=0}^{\infty}$, consistent with individual decisions and the stochastic processes for z and ξ, and given the tax subsidy τ, is characterized by the following implementability and feasibility conditions:*

$$w_t U_c(c_t, n_t) + U_n(c_t, n_t) = 0 \tag{1.25}$$

$$U_c(c_t, n_t) - \beta\left(\frac{R_t - \tau}{1 - \tau}\right) EU_c(c_{t+1}, n_{t+1}) = 0 \tag{1.26}$$

$$w_t n_t + b_t + d_t + c_t = \frac{b_{t+1}}{R_t} \tag{1.27}$$

$$F_n(z_t, k_t, n_t) = w_t\left(\frac{1}{1 - \mu_t \varphi_d(d_t)}\right) \tag{1.28}$$

$$Em_{t+1}\left(\frac{\varphi_d(d_t)}{\varphi_d(d_{t+1})}\right)\left[1-\delta+\Big(1-\mu_{t+1}\varphi_d(d_{t+1})\Big)F_k(z_{t+1},k_{t+1},n_{t+1})\right]+\xi_t\mu_t\varphi_d(d_t)=1 \tag{1.29}$$

$$R_tEm_{t+1}\left(\frac{\varphi_d(d_t)}{\varphi_d(d_{t+1})}\right)+\xi_t\mu_t\varphi_d(d_t)\left(\frac{R_t}{1+r_t}\right)=1 \tag{1.30}$$

$$(1-\delta)k_t+F(z_t,k_t,n_t)-w_tn_t+\frac{b_{t+1}}{R_t}-b_t-\varphi(d_t)+k_{t+1}=0 \tag{1.31}$$

$$\xi_t\left(k_{t+1}-\frac{b_{t+1}}{1+r_t}\right)\geq F(z_t,k_t,n_t) \tag{1.32}$$

$$TVC:\lim_{t\to\infty}E_t\beta^t\lambda_tk_t=0 \tag{1.33}$$

$$TVC:\lim_{t\to\infty}E_t\beta^t\lambda_tb_t=0 \tag{1.34}$$

$$TVC:\lim_{t\to\infty}E_t\beta^t\lambda_ts_t=0 \tag{1.35}$$

Where $R_t=1+r_t(1-\tau)$ and $\varphi(d_t)$ is the equity payout cost function.

Proof. In order to show this proposition, I need to show that conditions (1.25), (1.26), (1.27), (1.28), (1.29), (1.30), (1.31), (1.32), (1.33), (1.34) and (1.35) are necessary and sufficient for an equilibrium allocation $\{d_t,n_t,c_t,k_{t+1},b_{t+1},w_t,r_t\}_{t=0}^{\infty}$. Since the first three conditions (1.25), (1.26) and (1.27) are the first order conditions and budget constraint for households; conditions (1.28), (1.29), (1.30) and (1.31) are the first order conditions and the budget constraint for firms, where $m_{t+1}=\beta U_c(c_{t+1},n_{t+1})/U_c(c_t,n_t)$ is the effective discount factor; condition (1.32) is the enforcement constraint; and conditions (1.33), (1.34) and (1.35) are the transversality conditions associated with the firms and households' maximization problems, then it is straightforward that all these conditions are necessary and sufficient. QED

In order to show that issuing equity is equivalent to issuing and paying equity payout, it is necessary to compare the set of implementable allocations in each case and understand in which way those sets are related to each other [4]. First, it is important to prove that the set of allocations when $s_t = s_{t+1} = 1$ is contained in the set of allocations for all $s_t \in \Re^+$. It is also necessary to show that the optimal allocation in the larger set is an allocation in the setting $s_t = s_{t+1} = 1$, so that it follows that the optimal allocation is the same for both sets of allocations.

PROPOSITION 1. *The set of implementable allocations under* $s^*_{t+1} = 1$ *is a subset of the implementable set* $\left\{d^*_t, n^*_t, c^*_t, k^*_{t+1}, b^*_{t+1}, s^{**}_{t+1}, w^*_t, r^*_t, p^*_t\right\}_{t=0}^{\infty}$ *for any* $s^{**}_{t+1} \neq 1$.

Proof. Let $s_t = s^*_{t+1} = 1$. Then it implies that, in this case, the total amount of equity payout received from owning shares d'_t coincides with d_t, the total amount of dividends distributed to the shareholders in period t, i.e. $d'_t = d_t$. The equilibrium conditions for firms (1.8), (1.9) and (1.10), collapse to the ones given in Definition 2, i.e. conditions (1.28), (1.29) and (1.30). If we inspect the transformed conditions (1.15), (1.16) and (1.17), it is easy to confirm that these conditions are almost the same as conditions (1.28), (1.29) and (1.30), except that without the assumption that $s_t = s^*_{t+1} = 1$, we have $d'_t \neq d_t$. However, even without this assumption none of the conditions (1.8), (1.9) and (1.10) depend directly in equity shares s_t, only through the indirect influence of d'_t in the equity payout cost function. The budget constraint of the households and the firms also collapse to (1.27) and (1.31), respectively, when $s_t = s^*_{t+1} = 1$ is imposed. Therefore, there are policies that implement each allocation with $s_t = s^*_{t+1} = 1$. QED

Finally, we can show that the optimal allocation under $s_t = s^*_{t+1} = 1$ is the same as the optimal allocation for any $s^{**}_{t+1} \neq 1$. Then we have the following proposition:

[4]See Correia et al. (2008) for equivalence results and proofs on price-setting restrictions for the conduct of cyclical fiscal and monetary policy

PROPOSITION 2. *Let us assume that the set of implementable allocations:*

$\left\{d_t^*, n_t^*, c_t^*, k_{t+1}^*, b_{t+1}^*, s_{t+1}^* = 1, w_t^*, r_t^*, p_t^*\right\}_{t=0}^{\infty}$ *given* $\{\tau\}$, k_0, b_0 *and* s_0 *is an equilibrium. Then there is a large set of* $\left\{d_t^*, n_t^*, c_t^*, k_{t+1}^*, b_{t+1}^*, s_{t+1}^{**}, w_t^*, r_t^*, p_t^*\right\}_{t=0}^{\infty}$ *which are also equilibria.*

Proof. We need to show that the optimal allocation under the larger set $s_{t+1}^{**} \neq 1$ belongs to the set defined for any $s_t = s_{t+1}^* = 1$. Let's consider the problem of choosing a sequence of allocations, relative prices and policies $\left\{d_t^*, n_t^*, c_t^*, k_{t+1}^*, b_{t+1}^*, s_{t+1}^{**}, w_t^*, r_t^*, p_t^*\right\}_{t=0}^{\infty}$ that maximizes utility subject to households' budget constraint given in (1.27), and solve firms' optimization problem subject to the budget constraint and the enforcement constraint defined in (1.6), characterized by the conditions (1.21), (1.22), (1.15), (1.16) and (1.17). When we set $s_t = s_{t+1}^* = 1$, then, given the equity payout cost function defined as $\varphi(d_t') = d_t' + \kappa(d_t' - \overline{d})^2 = s_t d_t + p_t(s_t - s_{t+1}) + \kappa(s_t d_t + p_t(s_t - s_{t+1}) - \overline{d})^2$ and $\varphi_d(d_t') = s_t + 2\kappa s_t(d_t' - \bar{d}) = s_t + 2\kappa s_t(s_t d_t + p_t(s_t - s_{t+1}) - \bar{d})$, the equity payout cost function collapse to the original functional form presented in Jermann and Quadrini (2012):($\varphi(d_t) = d_t + \kappa(d_t - \overline{d})^2$). Given this, the firms' budget constraint become the one under $s_t = s_{t+1}^* = 1$, and since the enforcement constraint is common to both sets, then through Proposition 1 we know that the equilibrium conditions for firms (1.15), (1.16) and (1.17) collapse to the ones given in Definition 2, (1.28), (1.29) and (1.30), and the optimal solutions coincide. Thus, the set of optimal allocations under $s_{t+1}^{**} \neq 1$ also maximize welfare in the set of allocations under $s_t = s_{t+1}^* = 1$. QED

1.4.1.2 First Best

In order to better characterize the equilibrium, it is necessary to find the First Best allocation, or the allocation that maximizes the utility of the representative agent subject to the disposable technology and the resource constraint of the economy. In this case, it is not possible to define the Pareto Optimal equilibrium allocation of this economy, that is, the allocation such that there is no other allocation which some agents strictly prefer and which does not make any agents worse off, since this model deals with the representative agent problem, not the general equilibrium of the economy. Therefore, in order to find the First Best equilibrium of this economy, we need to solve the social planner's problem, which can be stated as follows:

$$\max_{\{c_{t+i}, n_{t+i}, k_{t+i}\}_{i=0}^{\infty}} \left\{ E_t \left[\sum_{i=0}^{\infty} \beta^i U(c_{t+i}, n_{t+i}) \right] \right\} \tag{1.36}$$

$$\begin{gathered} s.t. \\ c_t + i_t = F(z_t, k_t, n_t) \\ k_{t+1} = (1-\delta)k_t + i_t \\ c_t \geq 0 \\ k_0 \text{ given} \end{gathered}$$

That is, a benevolent social planner maximizes the households' utility function subject to the resource constraint of the economy, in which capital investment follows the law of motion defined previously. Given the presence of rational expectations (so that each period's decision takes into account new information acquired in that period), the marginal utility of wealth (or the shadow price) λ_t is allowed to vary across periods, being appropriately discounted. If we plug the capital law of motion into the resource constraint of the economy, we can rewrite the latter as $c_t + k_{t+1} - (1-\delta)k_t = F(z_t, k_t, n_t)$, and the Lagrangian of this problem is:

$$\mathcal{L} = E_t \left\{ \sum_{i=0}^{\infty} \beta^i \Big[U(c_{t+i}, n_{t+i}) + \lambda_{t+i}(F(z_{t+i}, k_{t+1}, n_{t+1}) - c_{t+i} - k_{t+1+i} + 1 - \delta) \Big] \right\} \tag{1.37}$$

Taking the necessary first order conditions with respect to the decision variables and the multiplier we get:

$$\lambda_t = U_c(c_t, n_t) \tag{1.38}$$

$$\beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \right) [F_k(z_{t+1}, k_{t+1}, n_{t+1})(1-\delta)] = 1 \tag{1.39}$$

$$F_n(z_t, k_t, n_t) = \frac{U_n(c_t, n_t)}{\lambda_t} \tag{1.40}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = z_t k_t^{\theta} n_t^{1-\theta} - c_t - k_{t+1} + (1-\delta)k_t = 0 \tag{1.41}$$

$$TVC : \lim_{\to\infty} E_t \beta^t \lambda_t k_t = 0 \tag{1.42}$$

From equation (1.38) we know that $\lambda_t = U_c(c_t, n_t) \Longrightarrow \lambda_{t+1} = U_n(c_{t+1}, n_{t+1})$, for any t. Plugging these in equation (1.39) generates the following condition:

$$1 = \beta E_t \left(\frac{U_c(c_{t+1}, n_{t+1})}{U_c(c_t, n_t)} \right) [1 - \delta + F_k(z_{t+1}, k_{t+1}, n_{t+1})] \tag{1.43}$$

From equations (1.38) and (1.40) we get the condition that represents the intertemporal choice between consumption and labor:

$$F_n(z_t, k_t, n_t) = \frac{U_n(c_t, n_t)}{U_c(c_t, n_t)} \tag{1.44}$$

We can retrieve the aggregate prices R, r, w and Em' from the firms' and the households' first order conditions only setting $\tau = 0$ and $\kappa = 0$, as it is shown in Proposition 4. From the firms' maximization problem we have:

$$\max_{\{n_t, k_{t+1}\}_{i=0}^{\infty}} \{\Pi_t = F(z_t, k_t, n_t) - w_t n_t - (R_t - (1-\delta))k_{t-1}\} \tag{1.45}$$

s.t.

$$z_t \sim H(z),$$

Where $H(z)$ is the distribution function for the stochastic productivity shock z_t. From the first-order conditions for this problem we get:

$$F_n(z_t, k_t, n_t) = w_t \tag{1.46}$$

$$\beta E_t[F_k(z_{t+1}, k_{t+1}, n_{t+1}) - R_t + (1-\delta)] = 0 \Leftrightarrow \tag{1.47}$$

$$\Leftrightarrow \beta E_t[F_k(z_{t+1}, k_{t+1}, n_{t+1}) + (1-\delta)] = R_t \tag{1.48}$$

Next, Proposition 3 identifies the implementable conditions and characterizes the set of implementable allocations of the First Best equilibrium, and Proposition 4 shows that the set of equilibrium allocations of the social planner's problem is the same set of allocations that define the First Best equilibrium only when the financial frictions are absent ($\tau = 0$ and $\kappa = 0$).

PROPOSITION 3. (Set of implementable allocations of the First Best allocation). *Assuming that the equity shares are set as $s_t = s_{t+1} = 1$, the set of implementable allocations, prices and policies $\{d_t, n_t, c_t, k_{t+1}, b_{t+1}, w_t, r_t\}_{t=0}^{\infty}$, consistent with individual decisions and the stochastic processes for z and ξ defined in the First Best equilibrium of this economy, is characterized by the following implementability and feasibility conditions:*

$$1 = \beta E_t m_{t+1} \left[1 - \delta + F_k(z_{t+1}, k_{t+1}, n_{t+1})\right] \tag{1.49}$$

$$F_n(z_t, k_t, n_t) = w_t \tag{1.50}$$

$$1 = R_t E_t m_{t+1} \tag{1.51}$$

$$c_t + k_{t+1} - (1-\delta)k_t = F(z_t, k_t, n_t) \tag{1.52}$$

where the stochastic discount factor is defined as

$$m_{t+j} = \beta E_t \left(\frac{U_c(c_{t+j}, n_{t+j})}{U_c(c_t, n_t)} \right) \tag{1.53}$$

Proof. I need to show that conditions (1.49), (1.50), (1.51), (1.52) and (1.53) are necessary and sufficient for an equilibrium allocation $\{d_t, n_t, c_t, k_{t+1}, b_{t+1}, w_t, r_t\}_{t=0}^{\infty}$. Conditions (1.49) and (1.53) are derived directly from the social planner's problem, and equation (1.52) is the resource constraint of the economy. Condition (1.50), which defines the labor demand, comes straightforward as a first order condition of the firm's problem (1.45), but additionally, if we combine the first order condition of the social planner's problem (1.44) and (1.46), we obtain that $\dfrac{U_n(c_t, n_t)}{U_c(c_t, n_t)} = w_t$. Finally, condition (1.51) results from the combination of equations (1.43) and (1.48), taking into account condition (1.53), relative to the stochastic discount factor. QED

PROPOSITION 4. (Implementability of the First Best allocation): *Assuming that the equity shares are set as $s_t = s_{t+1} = 1$, and that the social planner's equilibrium set of implementable allocations and policies $\{d_t, n_t, c_t, k_{t+1}, b_{t+1}\}_{t=0}^{\infty}$, is consistent with individual decisions and the stochastic processes for z and ξ, such that prices can be recovered from the firms's maximization problem, then that set of implementable allocations, prices and policies $\{d_t, n_t, c_t, k_{t+1}, b_{t+1}, w_t, r_t\}_{t=0}^{\infty}$ corresponds to the First Best allocation only if each allocation in this set of implementable allocations can be implemented with policies which set $\tau = 0$ and $\kappa = 0$.*

Proof. We must show that any implementable set of allocations $\{d_t, n_t, c_t, k_{t+1}, b_{t+1}\}_{t=0}^{\infty}$ defined for $s_t = s_{t+1} = 1$ in the competitive equilibrium with the implementation of a policy such that $\tau \neq 0$ *and* $\kappa \neq 0$ can only collapse to the First Best set of allocations when that policy is set to omit the financial frictions, i.e. $\tau = 0$ and $\kappa = 0$. We know, from Definition 2, that such set of allocations is characterized by conditions (1.25) until (1.32). If we set $\tau = 0$ and $\kappa = 0$ (which implies that the enforcement constraint no longer binds, i.e. $\mu = 0$, according to Proposition 5 and 6, then condition (1.26) and (1.30) collapse to condition (1.51), condition (1.28) collapses to condition (1.50), condition (1.29) collapses to condition (1.49), condition (1.25) becomes $\dfrac{U_n(c_t, n_t)}{U_c(c_t, n_t)} = w_t$, and the two budget constraints (1.27) and (1.31) (of the household and the firm respectively) combined collapse into the resource constraint of the economy $c_t + k_{t+1} - (1-\delta)k_t = F(z_t, k_t, n_t)$. Therefore, it is shown that any set of implementable allocations which characterize a competitive equilibrium for any $s_t = s_{t+1} = 1$ and where the financial frictions are absent ($\tau = 0$ and $\kappa = 0$) equals the set of implementable allocations which defines and characterizes the First Best equilibrium. QED

The First Best allocation in this framework can therefore be defined as follows:

DEFINITION 3 (First Best Equilibrium): *The First Best equilibrium can be decentralized as an allocation* $\{c_t(\boldsymbol{s}_t), n_t(\boldsymbol{s}_t), k_t(\boldsymbol{s}_t)\}_{t=0}^{\infty}$ *all* $\boldsymbol{s}_t$*; debt and equity levels* $\{b_t(\boldsymbol{s}_t), d_t(\boldsymbol{s}_t)\}_{t=0}^{\infty}$ *all* $\boldsymbol{s}_t$*; and prices and policies* $\{w_t(\boldsymbol{s}_t), r_t(\boldsymbol{s}_t), p_t(\boldsymbol{s}_t), m_t(\boldsymbol{s}, \boldsymbol{s}_{t+1})(\boldsymbol{s}_t)\}_{t=0}^{\infty}$ *all* $\boldsymbol{s}_t$*, characterized by a set of functions for the social planner's policies* $c^t(\boldsymbol{t})$*,* $n^t(\boldsymbol{t})$*,*$k^t(\boldsymbol{t})$*; (ii) firms' policies* $d(\boldsymbol{s}_t; k_t, b_t)$*,* $n(\boldsymbol{s}_t; k_t, b_t)$*,* $b(\boldsymbol{s}_t; k_t, b_t)$ *and* $s(\boldsymbol{s}_t; k_t, b_t)$*; (iii) aggregate prices* $w(\boldsymbol{s}_t)$*,* $r(\boldsymbol{s}_t)$*,* $p(\boldsymbol{s}_t)$ *and* $w(\boldsymbol{s}_t, \boldsymbol{s}')$*; (iv) law of motion for the aggregate states* $\boldsymbol{s}' = \Psi(\boldsymbol{s})$*, that solve the problems of the firms and the households, such that all markets clear: (i) households' policies satisfy conditions (1.20), (1.21) and (1.22) subject to the budget constraint given in (1.20); (ii) firms' policies are optimal and satisfy conditions (1.8), (1.9), (1.10) and (1.11) maximizing the utility function subject to the firm's budget constraint and the enforcement constraint; (iii) the wage and the interest rates clear the labor and bond markets through firms' conditions (1.8) and (1.10) and* $m(\boldsymbol{s}_t, \boldsymbol{s}_{t+1}) = \beta U_c(c_{t+1}, n_{t+1})/U_c(c_t, n_t)$*; (iv) the law of motion* $\Psi(\boldsymbol{s}_t)$ *is consistent with individual decisions and the stochastic processes for* z_t *and* ξ_t*.*

From all these conditions, we can conclude that the First Best solution corresponds to a standard RBC where business cycle fluctuations are only driven by the productivity/technological shock. In that framework, the enforcement constraint is not binding, and the steady state solution is not affected neither by the equity payout cost or by the tax advantage, which leaves firms indifferent in their choice of financing through debt or equity. In other words, it corresponds to an economy where the Modigliani-Miller proposition apply [5], i.e. an economy under which the total value (stocks plus debt) of a firm is independent of the firm's financial structure.

Given these conditions, it is necessary to identify the policy decisions affecting the financial frictions represented by the parameters τ and κ that can decentralize the First Best allocation of this model. It is shown in Proposition 5 and Proposition 6 below that such an allocation can only be decentralized through a competitive equilibrium when we set both $\tau = 0$ and $\kappa = 0$. These propositions are already defined by Jermann and Quadrini (2012), therefore I reproduce them here for convenience.

In Proposition 5 it is shown that for a deterministic steady state with constant z and ξ, the enforcement constraint is always binding. Proposition 6 shows that if there is no tax advantage of debt, i.e. $\tau = 0$ and there are no rigidities affecting the substitution between debt and equity, that is, $\kappa = 0$, changes in ξ have no effect on the real sector.

PROPOSITION 5: "*If $\tau > 0$ the enforcement constraint binds in a steady state.*

Proof. In a deterministic steady state we have that $m = 1/(1 + r)$ and $\varphi_d(d) = \varphi_d(d') = 1$. Therefore, the first order condition for debt, equation (1.10), simplifies to $REm + \bar{\xi}\mu R/(1+r) = 1$, where $\bar{\xi}$ is the average value. Substituting the above expression for m, we get $R/(1+r)+\bar{\xi}\mu R/(1+r) = 1$. Because $R = 1+r(1-\tau)$, this condition implies that $\mu > 0$ if $\tau > 0$. QED"

[5] See Modigliani and Miller (1958).

PROPOSITION 6: "*With $\tau = 0$ and $\kappa = 0$, changes in ξ have no effect on employment n and next period capital k'.*

Proof.When $\kappa = 0$ we have $\varphi_d(d) = \varphi_d(d') = 1$. Thus, the first order condition (1.4) can be written as $REm' + \xi\mu R/(1+r) = 1$. From the household's first order condition (1.7) we have $(1+r)Em' = 1$. Combining these two conditions we get $(1+\xi\mu)\dfrac{R}{1+r} = 1$, which implies that $\xi\mu = 0$ since $R = 1+r$ when $\tau = 0$. Therefore, μ is always zero and, assuming that the aggregate prices do not change, n and k' will not be affected by the change in ξ. We have to show next that the sequence of prices remains constant if firms do not change n and k'. This becomes obvious once we recognize that changes in debt issuance and equity payout associated with fluctuations in ξ cancel out in household's budget. Therefore, prices do not change. QED"

Therefore, according to this proposition, when $\tau = 0$ and $\kappa = 0$ the only shock that affects the economy is the productivity shock. This implies that the model resembles a standard RBC framework where firms indifferent between debt and equity, i.e., where the Modigliani-Miller theorem applies. Given that, according to these propositions, the First Best allocation can only be decentralized when $\tau = 0$ and $\kappa = 0$. In the next Section 1.4.1.3, I study different scenarios of the model for different values of τ and κ, assuming the presence or absence of the two shocks, separately and simultaneously, and justify, for each case, why the First Best allocation is not achieved, comparing the equilibrium allocations. Then, in that section, I also study the behaviour of the model assuming positive values for each one of the shocks (financial and productivity), and compare the equilibrium outcomes of each case with the First Best allocation.

1.4.1.3 Characterization with the financial frictions

This section studies how the two financial frictions of the model, the interest rate subsidy represented by τ and the equity payout cost function which depends on κ, are the major forces that prevent the equilibrium of this economy to achieve the First Best allocation.

First of all, it is important to understand how the First Best allocation can be decentralized in this model. It was shown in Proposition 5 and Proposition 6 that such an allocation can only be decentralized through a competitive equilibrium when we set both $\tau = 0$ and $\kappa = 0$.

For alternative values of τ and κ, however, these results are not necessarily true. Now I analyse the behaviour of this economy when we let these two parameters assume different positive values, first in an economy without random shocks, and then with it.

1.4.1.3.1 First Case: $\tau = 0$ and $\kappa > 0$ When we impose $\tau = 0$, although the steady state equilibrium always implies that $\mu = 0$ (that is, the enforcement constraint does not bind), it is not possible to achieve the First Best allocation in this situation, because outside the equilibrium the presence of the equity payout cost affects the stochastic discount factor, the firm's budget constraint and also the firm's first order conditions in relation to labor and capital. The fact that $\kappa > 0$ implies that it is costly for the firms to re-adjust their financial structure, which alone implies a larger volatility of the demand for labor and capital, respectively, but also induces a larger movement of the shadow price of the enforcement constraint μ, outside the steady state.

As it was already explained earlier in this paper, in order to better understand this mechanism, it is important to first consider the special case in which the cost of payout is zero, i.e., $\kappa = 0$. In this case $\varphi_d(d_t) = \varphi_d(d_{t+1}) = 1$ and condition (1.10) becomes $REm_{t+1} + \xi_t \mu_t R_t/(1 + r_t) = 1$, which implies that the Lagrange multiplier μ_t is fully determined by the aggregate prices R_t, r_t and Em_{t+1}. It is also clear from from conditions (1.8) and (1.9) that the production and investment choices of the firm (labor and capital choices, respectively) only depend on these prices. Taking as given R_t, r_t and Em_{t+1}, condition (1.10) with $\kappa = 0$ implies that there is a negative relationship between ξ_t and μ_t. This means that lower liquidation values of the firm's capital make the enforcement constraint tighter, i.e. the lower the probability that the lender can recover the full value k_{t+1}, the tighter is the enforcement constraint. Then, from condition (1.8) this negative relationship imposes that a higher μ implies a lower demand for labor. From condition (1.8) and (1.9) it is possible to see that the labor and capital investment choices of the firm only depend on aggregate prices. Changes in ξ_t only affect the investment policy of the firm given by condition (1.5) if they change the aggregate prices R_t and Em_{t+1}. But as long as the aggregate prices are not affected, the policy of the firm remains unchanged in these circumstances. It is also possible to observe that, in the absence of the equity payout cost, if the enforcement constraint is not binding in neither the current nor the next period, the Lagrange multiplier is $\mu_t = \mu_{t+1} = 0$. Then conditions (1.8) and (1.9) that determine the choice of labor and capital become $F_n(z_t, k_t, n_t) = w_t$ and $Em_{t+1}[1 - \delta + F_k(z_{t+1}, k_{t+1}, n_t)] = 1$, that is, the marginal productivities equal their marginal costs, just like in a standard real business cycle framework, which constitutes the benchmark of this model.

When $k > 0$, this mechanism that establishes a negative relation between ξ_t and μ_t is reinforced, since in that case it is costly to re-adjust the financial structure, and the change in ξ_t induces a larger movement in μ_t. In other words, a fall in the liquidation value of the firm's capital (a negative financial shock) will make the enforcement constraint tighter (μ_t increases). In the first period t after the shock, this will lead to a reduction in the demand of labor, since the increases in μ increases the marginal cost of labor, as it can be observed in condition (1.8). Then, starting from the next period $t+1$, the input of capital will also decrease (see condition (1.9)). Furthermore, in equilibrium changes in the financial policies of all firms also affects aggregate prices prices R_t and m_{t+1}, with some feedbacks on individual policies.

In the steady state, and assuming whether or not that there is no interest rate benefit (i.e. $\tau = 0$), having $\kappa = 0$ or a positive value for κ does not affect the equilibrium conditions, because in this case we always have $\varphi_d(d_t) = \varphi_d(d_{t+1}) = 1 + 2\kappa(d_t - \bar{d}) = 1 + 2\kappa(d_{t+1} - \bar{d}) = 1$, since $d_t = d_{t+1} = \bar{d}$ must hold in the stationary equilibrium. Hence, in these circumstances, the steady state calibrated value of κ is irrelevant to determine changes on the steady state level of other variables of the model.

Dynamically, however, the simulation of the model for different values of κ confirms the mechanism between the value of κ, ξ and μ. Figure 1.1 plots the impulse response functions (IRF's) to the financial shock, for a model simulated with both shocks and with both frictions ($\tau > 0$ and $\kappa > 0$) and for the model simulated with both shocks and the interest rate benefit but without the equity payout cost ($\tau > 0$ and $\kappa = 0$). Figure 1.2 displays the same comparison between the impulse responses of a one-time productivity shock of those two simulated models.

From Figure 1.1, in response to a positive shock to ξ_t (a negative financial shock, since $1 - \xi_t$ is the probability that the recovery value is zero, and therefore, represents the financial shock as a phenomenon with negative effects over the economy), the impulse responses of most of the variables are much greater when the equity payout cost in included in the model ($\kappa > 0$). This fact is specially true for the multiplier μ and the number of hours worked (n_t), as predicted by the mechanism described above, which predicts that, given the negative connection between the innovations in ξ_t and the shadow price of the enforcement constraint μ_t, the volatility and the impulse response of μ_t is much stronger the higher is the weight of the equity payout cost, as measured by κ. As a consequence, the impulse responses to the financial shock for labor, capital, output, consumption and also the aggregate prices R_t and w_t are much higher when the equity payout cost is present. The only exceptions are the impulse responses for debt repurchases and equity payout, whose impulse responses to the financial shock become weaker when $\kappa > 0$. One possible explanation is that with the presence of the equity payout cost, re-adjustments of the financial structure are costly and slower for firms to readjust, and therefore, the impulse responses of debt and equity payout to the financial shock are smoother.

In relation to the productivity shock, these differences in the sign and the magnitude of the impulse response functions are not that larger as in the financial shock case, as can be observed in Figure 1.2. For $\kappa = 0$ all the impulse responses are higher than in the case where the equity payout cost is present (and calibrated as $\kappa = 0.146$), for most variables, except for debt repurchases b_t and the Lagrange multiplier μ_t. These impulse responses confirm the mechanism that operates when a positive productivity shock occurs: firms increase their investment and raise its capital stock in the subsequent periods after the shock to take advantage of higher expected productivity, which has an amplification effect over output, consumption, the wage rate and the gross interest rate. Debt repurchases decrease (i.e. the outstanding debt of firms increases) and the equity payout falls since firms prefer to increase their leverage and take advantage of higher productivity to finance future payments to shareholders, instead of raising the equity payout and dividends. However, these effects occur when the equity payout cost is absent, but they are substantially reduced when $\kappa > 0$. In that case, the cost of adjusting equity is high, which makes the impulse response for equity payout much lower in magnitude and smoother over time, and inducing firms to reduce the issuance of new debt, as can be observed in the impulse response of debt repurchases in Figure 1.2.

However, the effect of the productivity shock over hours worked has a more interesting behaviour when the equity payout cost is present. In fact, when it is expected a higher productivity, then the marginal productivity of labor is also expected to increase, raising its demand, as can be observed in condition (1.8). The marginal cost of labor is lower when the equity payout is absent, since the wedge that augments the wage rate only depends on the shadow price of the enforcement constraint, μ_t, in that case. Therefore, when the productivity shock hits the economy and the marginal productivity of labor rises, the difference to the marginal cost is greater when $\kappa = 0$, and so the impulse response of hours worked to this shock is positive and far greater than in the case where the equity payout cost is absent. Therefore, to a certain extent, the effect of the productivity shock over employment depends on the trade-off between debt and equity when the firm has to decide the composition of its financial structure.

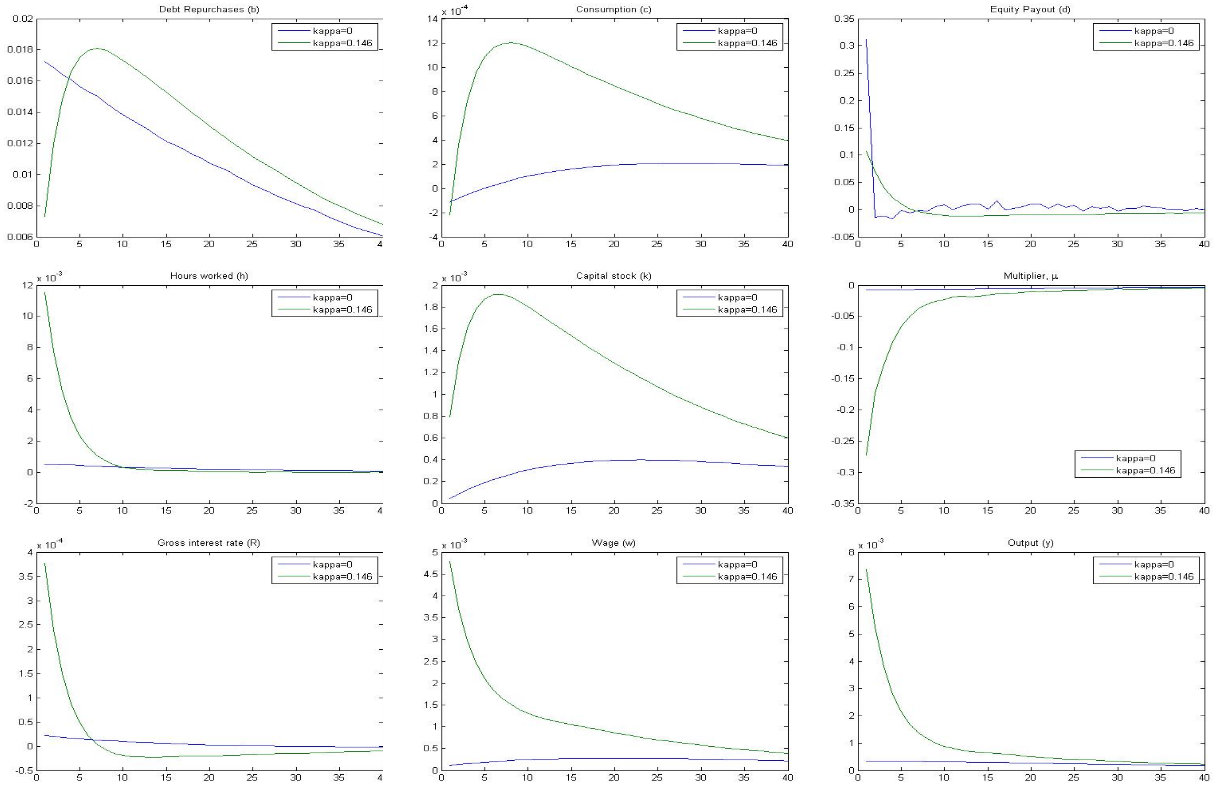


Figure 1.1: Impulse Responses to one-time Financial Shock with and without the Equity Payout Cost

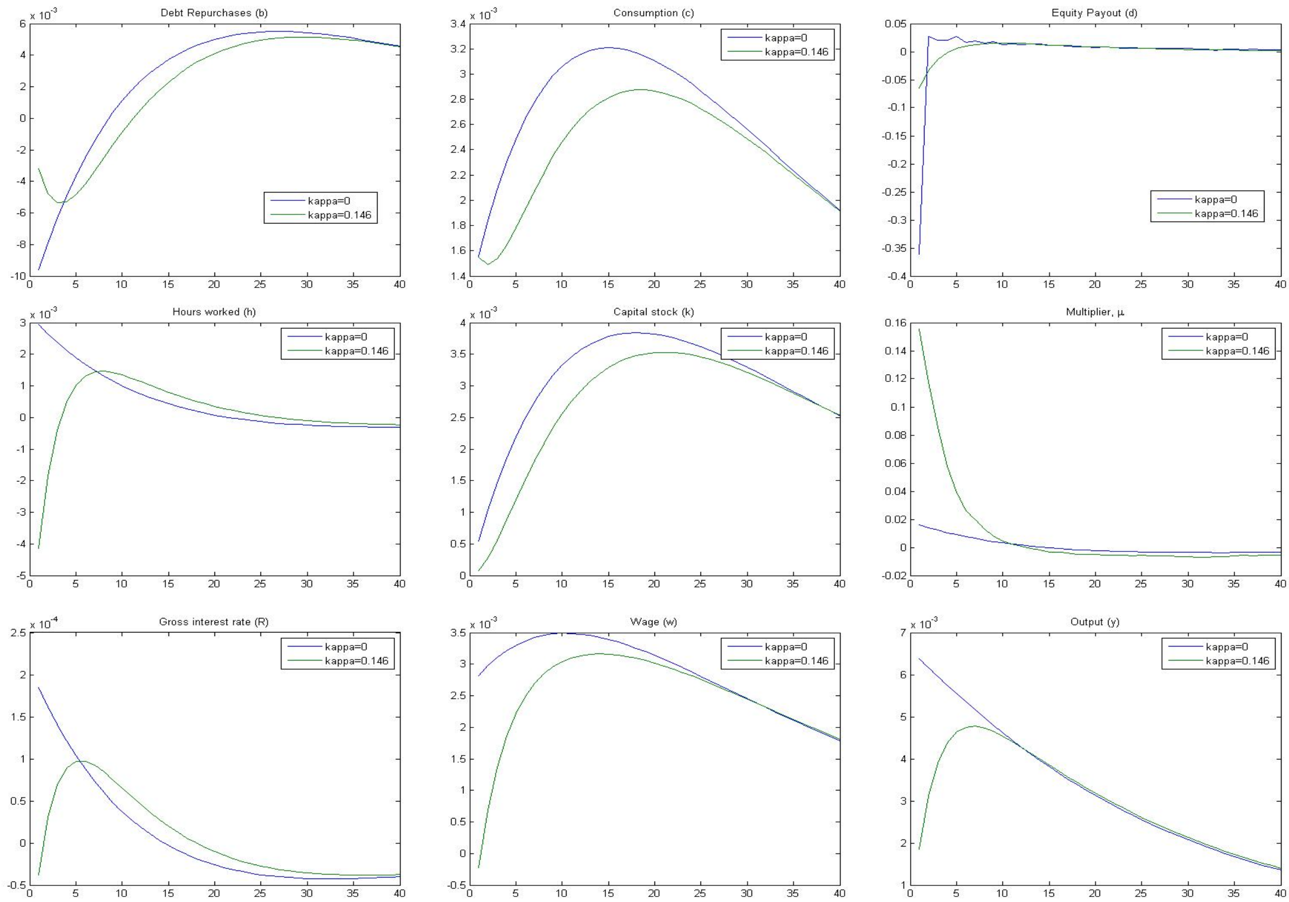


Figure 1.2: Impulse Responses to one-time Productivity Shock with and without the Equity Payout Cost

1.4.1.3.2 Second Case: $\tau > 0$ and $\kappa = 0$ The presence or absence of the interest rate benefit is crucial in the model, because it can cause serious changes at the steady state level which can bias completely the welfare analysis of the model. The parameter τ is crucial for the quantitative performance of the model because it determines whether or not the enforcement constraint is binding, as it was shown in Proposition 5. In fact, in the deterministic steady state of this economy we have $\bar{\mu} = \dfrac{\tau(1-\beta)}{1-\tau(1-\beta)}$, and hence, the enforcement constraint becomes tighter in the steady state as the value of the tax benefit τ increases. This has important consequences in the steady state variables of the model: as the value of τ increases and the enforcement constraint becomes tighter, the majority of the variables (consumption, capital, output, debt repurchases, equity payout and wages) increase; except for the gross interest rate R_t, which decreases with τ (in the steady state it is determined by the condition $\bar{R} = \frac{1}{\beta}(1-\tau)1-\beta)))$, and hours worked, which decreases for values of $\tau \leq 0.35$, and increases for $0.35 < \tau \leq 1$.

As can be observed in Table 1.1 (and with more detail in Table 1.9, in Section 1.7.5 of the appendix), as the tax benefit increases and the enforcement constraint gets tighter, the reduction in the gross interest rate boosts investment, and therefore capital increases, but it is also easier to borrow, and debt also raises. For some variables an increasing τ has amplification effects, not only because some of the variables depend directly on τ in the steady state with a positive sign, but also because depend positively on the capital steady state level. These are the cases of output, consumption,wages and equity payout. However, the increase in the steady state levels of those variables is not as strong as in the case of capital and debt because there are some opposing effects that cancel part of this boost effect. This is specially true for hours of work and consumption.

All these steady state level changes occur despite the fact that the interest rate benefit is financed through a lump-sum tax over households.

These drastic shifts in the steady state levels of variables when we change the value of τ obviously also produce dramatic differences in the behaviour of the steady state period utility and in the expected utility of the model. In Table 1, the first and second columns show that the steady state utility level per period and the expected utility increase as τ increases, i.e., the welfare of households in the economy increases as this distortion (a tax benefit for firms) becomes tighter, which is a highly counter-intuitive result.

This problem arises since including or excluding the tax benefit affects the steady state levels of the variables, including the steady state utility for each period. Therefore, it is necessary to make corrections in the welfare costs calculations that take into account this shift in the steady state levels of the variables, in order to avoid serious biases in the results. The procedure that was followed to correct that bias considers the model without the tax benefit (i.e. $\tau = 0$) as the steady state benchmark, and will be fully explained in Section 1.6.1, in which the methodology to compute the welfare costs of the shocks is presented.

The expected utility results, already corrected for this bias in the deterministic steady state, are also reported in the third column of Table 1.1. It can be observed that, although there is a slight increase when we pass from the absence of the tax benefit ($\tau = 0$) to a small positive value ($\tau = 0.1$), the expected utility decreases as the value of τ also increases, as it is expected in the presence of a tax distortion in the economy.

Table 1.1: **The Effect of the Tax Subsidy (τ) in Steady State**

τ	**SS Period Utility**	**Expected Utility (NC)**	**Expected Utility (C)**	c^{SS}	n^{SS}	k^{SS}	b^{SS}	μ^{SS}
0	-0.913486	-52.1992	-52.2217	0.78702	0.30083	8.37959	2.32161	0.00000153
0.1	-0.904527	-51.6873	-52.2192	0.79314	0.30037	8.79302	2.63992	0.01072824
0.2	-0.89531	-51.1606	-52.2193	0.79985	0.30008	9.25907	2.99990	0.02148352
0.35	-0.881045	-50.3454	-52.2195	0.81117	0.30000	10.07974	3.63684	0.03766313
0.4	-0.876192	-50.0681	-52.2197	0.81532	0.30009	10.39236	3.88049	0.04306763
0.5	-0.866381	-49.5075	-52.2201	0.82424	0.30049	11.08893	4.42543	0.05389145
0.6	-0.856512	-48.9435	-52.2205	0.83406	0.30123	11.89910	5.06263	0.06473193
0.7	-0.846721	-48.3841	-52.2214	0.84489	0.30238	12.85136	5.81603	0.07558454
0.8	-0.837225	-47.8414	-52.2230	0.85683	0.30406	13.98423	6.71816	0.08644333
0.9	-0.828354	-47.3345	-52.2277	0.869999	0.30641	15.35098	7.81426	0.09730007

NOTE: SS - Steady State; NC - Not Corrected; C - Corrected; it is presented the results for $\tau = 0.35$ instead of $\tau = 0.3$ because the former is the value used as the steady state target in the original calibration of Jermann and Quadrini (2012).

1.4.1.4 Characterization with the financial shock

In this section, I consider the model with only the financial shock when $\tau = 0$ and $\kappa = 0$, in order to compare it with the First Best equilibrium allocation. In this case, from Proposition 6, we know that innovations in the financial shock ξ have no effect on the real side of the economy, particularly over employment n and next period capital stock k_{t+1}. Therefore, assuming that the financial shock would be the only source of uncertainty would be equivalent to assume that there was randomness at all in this economy, and the model would therefore resemble a frictionless deterministic economy, without any dynamic transition from one period to another. In this version of the model the financial flows become indeterminate because firms are indifferent between debt and equity financing (the Modigliani-Miller theorem applies in this case). In order to illustrate this statement, it can be observed in Table 1.10 of Section 1.7.6 of the Appendix, that the expected utility of the model with only the financial shock (model 10) equals the expected utility of the frictionless model (model 16), when $\tau = 0$ and $\kappa = 0$. This implies that the welfare cost of these two models in relation to the benchmark economy (the model with only the productivity shock when $\tau = 0$ and $\kappa = 0$ (Model 6) must be the same. This result will be confirmed in Section 1.6.3, where the findings of the welfare costs analysis are presented.

1.4.1.5 Characterization with both shocks

Maintaining the assumption that $\tau = 0$ and $\kappa = 0$, hitting this economy with both the financial and the productivity shocks simultaneously is equivalent to the First Best solution for this model, which corresponds to the economy without frictions with only the technological shock. As it was shown in Propositions 5 and 6 this is true because, as long as $\tau = 0$ and $\kappa = 0$, the financial shock has no real effects over the economy (particularly over employment n_t) and next period capital k_{t+1} and therefore, business cycle fluctuations are only driven by productivity shocks. This implies that when $\tau = 0$ and $\kappa = 0$, in this framework, the model with both shocks (model 4) is equivalent to the benchmark model with only the productivity shock (model 6), in terms of welfare effects in the economy. As can be seen from Table 1.10 of Section 1.7.6 of the Appendix, the expected utilities of both models are very close, which will generate very close welfare costs, as Section 1.6.3 will show.

1.5 Empirical analysis

In order to evaluate the quantitative effects of the productivity and the financial shocks over the real economy, the approach followed by Jermann and Quadrini (2012) is based on the construction of time series using the standard Solow residuals approach for the productivity shocks and using the enforcement constraint for the financial shock. The macroeconomic effects are then captured by the responses of the model to the shocks. In this simulation, two points are important to emphasize: first, the finding that financial shocks have played an important role in the U.S. business cycle does not mean that other shocks are not relevant; and second, the fact that this simulation do not include other shocks apart from the productivity and the financial shocks does not bias the results since the approach used to identify the financial shocks is independent of how many shocks are added to the model.

I follow exactly the same procedure as Jermann and Quadrini (2012) in order to calibrate the model, and for convenience, I reproduce it here:

1.5.1 Parameterization

"The parameters are grouped into two sets: the first set includes parameters that can be calibrated using steady state targets, some of which are common in the real business cycle literature; and the second one includes parameters that cannot be calibrated using steady state targets. Time periods are measured in quarters."

1.5.1.1 Parameters set with Steady State targets

"In the case of the parameters set with steady state target, I set $\beta = 0.9825$, implying that the annual steady state return from holding shares is 7.32 percent, according to Jermann and Quadrini (2012) estimations. The utility function has the functional form $U(c, n) = ln(c) + \alpha.ln(1 - n)$, where $\alpha = 1.8834$ is chosen to have steady state hours equal to 0.3. The Cobb-Douglas parameter in the production function is set to $\theta = 0.36$ and the depreciation to $\delta = 0.025$. The mean value of z is normalized to 1. These values are standard and the quantitative properties of the model are not very sensitive to this first group of parameters.

The tax wedge is set to $\tau = 0.35$, which corresponds to the benefit of debt over equity if the marginal tax rate is 35 percent. This parameter is very important for the model because it determines whether the enforcement constraint is binding or not. In fact, this value of τ and the all the remaining parameterizations of the model are set in order to make the enforcement constraint always binding in the simulations (except when we set $\tau = 0$ in order to simulate the model without the interest rate benefit).

The mean value of the financial variable, $\bar{\xi}$, is chosen to have a steady state ratio of debt over quarterly GDP equal to 3.36. this is the average ratio over the first quarter of 1984 until the second quarter of 2010 for the nonfinancial business sector based on data from the Flow of Funds (for debt) and National Income and Product Accounts (for business GDP). The required value is $\bar{\xi} = 0.1634$."

1.5.1.2 Parameters that cannot be set with Steady State targets

"The parameters that cannot be set with steady state targets are those determining the stochastic properties of the shocks and the cost of equity payout - the parameter κ. In a steady state the stochastic properties of the shocks do not matter and the equity payout is always equal to the long-term target, therefore an alternative procedure was followed to construct the series of productivity and financial shocks.

For the productivity variable z_t it was used the standard Solow residuals approach. Using the production function we get

$$\hat{z}_t = \hat{y}_t - \theta \hat{k}_t - (1-\theta)\hat{n}_t \tag{1.54}$$

where $\hat{z}_t$, $\hat{y}_t$, $\hat{k}_t$ and $\hat{n}_t$ are the percentage or log-deviations from the deterministic trend. Using the calibration for θ and the empirical series for $\hat{y}_t$, $\hat{k}_t$ and $\hat{n}_t$, we construct the $\hat{z}_t$ series.

To construct the series for the financial variable ξ_t, we follow a similar approach but using the enforcement constraint under the assumption that it is always binding, that is:

$$\xi_t \left(k_{t+1} - \frac{b_{t+1}}{1+r_t} \right) = y_t \tag{1.55}$$

The variable ξ_t is determined residually using empirical series for k_{t+1}, $b_{t+1}/(1+r_t)$ and y_t. However, the validity of this procedures depends crucially on the validity of the assumption that the enforcement constraint is always binding."

Jermann and Quadrini (2012) have verified ex-post, after constructing the series for the shocks and feed them into the model, whether the enforcement constraint in always binding, but without forcing any of the endogenous variables to perfectly match an individual empirical series.

"The data series of capital k_{t+1} and debt $\dfrac{b_{t+1}}{1+r_t}$ are end-of period balance sheet data from the Flow of funds Accounts, and the empirical series for the product y_t is taken from the national Income and Product Accounts (NIPA). All series are in real terms and the log value is linearly detrended."

In order to compute the processes for the two shocks, Jermann and Quadrini (2012) have estimated a vector autoregressive system of order one (VAR(1)), after constructing the series for the productivity and financial variables over the first quarter of 1984 until the second quarter of 2010:

$$\begin{pmatrix} \hat{z}_{t+1} \\ \hat{\xi}_{t+1} \end{pmatrix} = \mathbf{A} \begin{pmatrix} \hat{z}_t \\ \hat{\xi}_t \end{pmatrix} + \begin{pmatrix} \epsilon_{z,t+1} \\ \epsilon_{\xi,t+1} \end{pmatrix} \tag{1.56}$$

where $\epsilon_{z,t+1}$ and $\epsilon_{\xi,t+1}$ are iid with standard deviations σ_z and σ_ξ, respectively.

From this description of the procedure used to construct the series for the productivity and the financial shocks, the fact that "these series do not depend on the number of shocks included in the model becomes more clear. Since no matter how many shocks we include in the model, equations (1.54) and (1.55) will not be affected". Jermann and Quadrini (2012) state that "the only way an additional shock could affect the ξ_t series is in the eventuality that it could change the tightness of the enforcement constraint. With additional shocks, one cannot guarantee that the enforcement constraint will be always binding in the simulated period". Jermann and Quadrini (2012) consider it unlikely with the typical shocks considered in the literature. However, they extend the model following the Smets and Wouters (2007) approach, and use a structural estimation with eight shocks (besides the productivity and the financial shocks) to simulate the model and check whether the inclusion of more shocks affects substantially the main results of the model.

The only remaining parameter is the equity cost parameter κ. This is chosen to have a standard deviation of equity payout (normalized by output) generated by the model over the same period of time 1984.I-2010.II equal to the empirical standard deviation.

The full set of parameters are reported in Table 1.2.

Table 1.2: Parameterization

Description	**Parameters**
Discount factor	$\beta = 0.9825$
Tax advantage	$\tau = 0.3500$
Utility parameter	$\alpha = 1.8834$
Production technology	$\theta = 0.3600$
Depreciation rate	$\delta = 0.0250$
Enforcement parameter	$\bar{\xi} = 0.1634$
Payout cost parameter	$\kappa = 0.1460$
Standard deviation productivity shock	$\sigma_z = 0.0045$
Standard deviation financial shock	$\sigma_\xi = 0.0098$
Matrix for the shocks process	$\mathbf{A} = \begin{bmatrix} 0.9457 & -0.0091 \\ 0.0321 & 0.9703 \end{bmatrix}$

Source: Jermann and Quadrini (2012)

1.5.2 Estimation

Since the model cannot solved analytically, I used numerical methods in order to estimate this model. In this case, the model is solved using a linear approximation of the dynamic system under the assumption that the enforcement constraint is always binding. To check the validity of this assumption and to check the accuracy of the linear solution, Jermann and Quadrini (2012) also solved the model nonlinearly using a global approximation method.

Taken into account the assumption that the enforcement constraint is always binding, the model can be solve by log-linearizing the dynamic system around the steady state. The equilibrium in the base model is characterized by a system composed of 8 dynamic equations:

$$wU_c(c,n) + U_n(c,n) = 0 \tag{1.57}$$

$$U_c(c,n) = \beta\left(\frac{R-\tau}{1-\tau}\right)EU_c(c',n') \tag{1.58}$$

$$wn + b - \frac{b'}{R} + d - c = 0 \tag{1.59}$$

$$F_n(z,k,n) = w\left(\frac{1}{1-\mu\varphi_d(d)}\right) \tag{1.60}$$

$$E\tilde{m}(c,n,d,c',n',d')\Big[1-\delta+(1-\mu'\varphi_d(d'))F_k(z',k',n')\Big]\xi\mu\varphi_d(d) = 1 \tag{1.61}$$

$$RE\tilde{m}(c,n,d,c',n',d') + \xi\mu\varphi_d(d)\left(\frac{R(1-\tau)}{R-\tau}\right) = 1 \tag{1.62}$$

$$(1-\delta)k + F(z,k,n) - wn - b + \frac{b'}{R} - k' - \varphi(d) = 0 \tag{1.63}$$

$$\xi\left(k' - b'\frac{1-\tau}{R-\tau}\right) = F(z,k,n) \tag{1.64}$$

According to Jermann and Quadrini (2012)' approach, "the first three equations are the first order conditions and budget constraint for households. In equilibrium the tax payments of households is accounted by a lower interest earned on bonds, R, and the gross pre-tax interest rate is $1 + r = (R - \tau)/(1 - \tau)$. The next three equations are the first order conditions for firms. The term $\tilde{m}(c, n, d, c', n', d') = \beta(U_c(c', n')/U_c(c, n))(\varphi_d(d)/\varphi_d(d'))$ is the effective discount factor. The remaining two equations are the firms' budget and enforcement constraints.

After linearizing around the steady state the system can be resolved for the eight variables $c_t, d_t, n_t, w_t, R_t, \mu_t, k_{t+1}, b_{t+1}$, as linear functions of the states, z_t, ξ_t, k_t, b_t."

1.5.3 Results and Analysis

In order to study the effects induced by the productivity and the financial shocks, sixteen simulations of the model were run: four major simulations depending on the inclusion or absence (simultaneous or not) of the productivity and the financial shocks; and then for each one these four cases, four more simulations depending whether the financial frictions set by the parameters τ and κ are active (simultaneously) or not. In the model with only the financial shock active, the productivity shock is kept constant at its unconditional mean $\bar{z}$, and the same procedure is followed when the model with only the productivity shock is active (the financial variable ξ_t is kept constant at its unconditional mean $\bar{\xi}$. The full list of all the sixteen simulated models is in Section 1.7.3 of the Appendix.

After simulating the model, I give a brief description of the main findings achieved by Jermann and Quadrini (2012). But before start describing the behavior or the simulated variables and comparing it with the data, it is very important to stress two important aspects about the model: first, "the macroeconomic effects of financial shocks in this model are mostly driven by the unexpected "variations" in ξ_t, not the "level" of this variable". Although a low value of ξ_t may produce moderate variability on hours and investment, if the decrease has not taken place recently (i.e. if the economy had time to adjust to the lower ξ_t), it is the change that matters, not the level of the variable. It is in this sense that the 2008 financial crisis is characterized by the most severe financial conditions experienced by the US economy during the last three decades. The second aspect is the fact that "the time series constructed for the shocks do not depend on the number of shocks included in the model, which implies that, given empirical series for k_{t+1}, $b_{t+1}/(1 + r_t)$ and y_t, the exact same series for the financial shocks ξ would be generated".

Their main findings suggest that the performance of the model improves with the inclusion of the financial shock, which captures the firm's ability to borrow as well as tight financial conditions. The model with financial shocks is able to better capture the dynamics of output, labor and the financial variables than the model with productivity shocks only. With financial shocks only, the dynamics of the major macroeconomic variables (such as output and labor) fit very well to the data. For these variables, the financial innovations of the simulated model generate sharp declines in all three recessions registered in the U.S in the last decade: 1990-91, 2001 and 2008-09. Jermann and Quadrini (2012) justify this performance of the model in response to financial shocks based on the mechanism which links these shocks to the demand of labor, since they generate a high volatility in working hours. As it was explained earlier in Section 1.3.1, the strong correlation between financial shocks and hours of work can be seen from the demand of labor given by the firm's first order condition (1.3), where the variable μ is the Lagrange multiplier associated with the enforcement constraint and the term $\mu\varphi_d(d)$ determines the labor wedge. If the economy is hit by a negative financial shock, which makes the enforcement constraint tighter (μ increases), the labor wedge also increases, increasing the marginal cost of labor, reducing the demand of working hours. Jermann and Quadrini (2012) argue that, intuitively, if the firm wants to keep the same scale and hire the same number of workers, it has to reduce equity payout, since the marginal cost of each worker has risen. Due to the equity payout cost, that reduction is costly and it is not immediate, therefore the firm chooses to sacrifice both the equity payout and in part the input of labor.

The model with financial shocks is also able to capture the main features of the empirical series of the financial flows, debt and equity payout, although the volatility of debt repurchases generated by the model is somewhat higher than in the data.

In general, Jermann and Quadrini (2012) conclude that the model with both the financial and the productivity shocks is able to replicate reasonably well the dynamics of the major real variables included in the model (specially output and labor) as well as the dynamics of the financial variables debt repurchases and equity payout.

1.6 The Welfare Cost of Financial Shocks

1.6.1 Estimation Methodology

This section constitutes the main focus of this paper, since it describes how the welfare costs of both shocks are computed and also reports the main results. In particular, it is reported the isolated welfare cost associated exclusively with the financial shock.

In order to compute these potential welfare costs of financial shocks, I follow Lucas' strategy (1987, 2003) to compare the welfare effects of implementing a stabilization policy aimed at eliminating all variability caused not by consumption fluctuations, but instead by the financial shock. In other words, I compute the percentage increase in consumption that an agent would require to be as well off as under the First Best allocation of this economy. In this case, the First Best allocation is equivalent to the equilibrium allocation of this economy without the enforcement constraint ($\tau = 0$, i.e. the financial shock has no real effects and there is no substitutability between debt and equity, i.e. $\kappa = 0$). This translates into a comparison between the utility derived from a consumption stream c_t^F associated with the model simulated with both frictions and the financial shock, and the utility derived from a consumption stream c_t^{FB}, associated with the standard model calibrated with only the productivity shock and no frictions, i.e. the First Best equilibrium. In other words, in this model, the welfare cost of the financial shock is the percentage change in the consumption of an agent of the economy hit by the shock necessary to achieve the same level of utility attained by an agent in the First Best economy. These utilities are both computed in steady-state. Assuming that the latter is preferred to the former, we have:

$U(c_t^F) < U(c_t^{FB})$

where F stands for the economy simulated with only the financial shock and FB represents the First Best equilibrium.

To quantify the potential welfare gains or costs that can result in moving from F to FB, we can proceed as follows:

$$U^F(\xi_t > 0, (1+\lambda)c_t^F, n_t) = U^{FB}(\xi_t = 0, c_t^{FB}, n_t) \tag{1.65}$$

This way, welfare costs are measured in percentage units of the level of consumption realized under economy F. Since both economies considered are not deterministic but stochastic, preferences must be computed as expected utilities, and the expectation is taken with respect to the common distribution of the shocks ξ_t and z_t, for all t. The functional form for the utility function is $U(c_t, n_t) = ln(c_t) + \alpha ln(1 - n_t)$, and this implies that the welfare comparison involving the calculation of λ can be written as:

$$E_0 \sum_{t=0}^{\infty} \beta^t U^F(\xi_t > 0, (1+\lambda)c_t^F, n_t) = E_0 \sum_{t=0}^{\infty} \beta^t U^{FB}(\xi_t = 0, c_t^{FB}, n_t) \Leftrightarrow \tag{1.66}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t ln((1+\lambda)c_t^F) + \alpha ln(1 - n_t^F) = E_0 \sum_{t=0}^{\infty} \beta^t ln(c_t^{FB}) + \alpha ln(1 - n_t^{FB}) \tag{1.67}$$

Now in order to determine c_t^F and c_t^{FB} in steady-state, it is necessary to calibrate and simulate both models and use the generated series to compute λ. However, this process is not as simple as it may seem, since it is necessary to overcome some technical problems that arise when the steady state simulated values for each variable are computed.

However, first of all, before presenting the complete description of the methodology used to compute the welfare effects in this model, it must be stressed I report all the welfare costs for each type of economy that can be simulated in this framework, depending of the presence and/or absence of the shocks (financial and technological) and the financial frictions (the tax subsidy and the equity payout cost), in comparison with the First Best equilibrium (i.e. the economy without financial frictions and with the productivity shock) and also in comparison to a frictionless economy (i.e. the model simulated with no shocks and no frictions). This ensures a complete welfare overview of this model considering all possible scenarios.

In order to achieve that goal, I performed simulations for sixteen different types of economies that can be considered assuming different combinations of inclusion/exclusion of the shocks and the financial frictions present in this framework. I considered four base types of model to estimate: the economy without shocks (WS); the economy with only the financial shock (F); the economy with only the productivity shock (P); and the economy with both shocks (BS). Then, for each one of these different base models, I considered the presence or omission of the two financial frictions: the interest rate subsidy (τ) and the equity payout cost function (κ). Table 1.8 in Section 1.7.3 of the Appendix gives a description of all those different simulated economies and the correspondent numerical order by which each model was simulated.

Given the set of parameter values described earlier and in order to consider changes in policy and take into account the effects of the transitional dynamics, the next step was to run 120 simulations with 100000 periods each for the sixteen different models, using a different initial seed as a starting point for each simulation. The simulation is done using a linear first-order approximation of the calibrated model. From each one of these simulations, I obtained series of simulated values for c and n, from where we can compute a stream for the utility U, for $t = 100000$ periods :

$$U^i = E_0 \sum_{t=0}^{\infty} \beta^t ln(c_t^i) + \alpha ln(1 - n_t^i) \tag{1.68}$$

for $i = F, P, FB, BS$

The economy without any type of shock corresponds to a frictionless, stationary economy, where the simulated values for all variables are the same for all periods, and therefore constitutes the deterministic steady state of this model. Hence, its expected value can be computed as the sum of infinite terms of a geometric series (β^i), where the period utility $U^{WS} = ln(c_t^{WS}) + \alpha ln(1 - n_t^{WS})$ is constant across all time periods. Hence, we have:

$$U^{WS} = E_0 \sum_{t=0}^{\infty} \beta^t ln(c_t^{WS}) + \alpha ln(1 - n_t^{WS}) \Leftrightarrow \tag{1.69}$$

$$\Leftrightarrow U^{WS} = \frac{ln(c_t^{WS}) + \alpha ln(1 - n_t^{WS})}{1 - \beta} \tag{1.70}$$

After simulating all the models, I computed the expected lifetime utility for each simulation over the 100000 periods, and then I calculated the weighted average of the expected utilities across the 120 different simulations, in order to find the expected utility for each one of the 16 different economies.

However, after inspecting the estimates of those expected utilities for the different sixteen simulated economies presented in Table 1.10 in the Appendix, we must conclude that there are very significant differences in the level of those estimates. This is essentially due to the presence or omission of the tax benefit τ. From Proposition 5 we know that if $\tau > 0$ the enforcement constraint binds in a steady state, and that implies that changing the value of τ causes major differences in the steady state levels of the variables of the simulated models, which also affects steady state period utility levels. This constitutes a problem for the welfare analysis of the model, since it produces biased welfare costs estimates.

Therefore, a correction procedure in the calculation of the expected utilities for the different sixteen economies was implemented in order to overcome this problem. Since the utility estimates depend directly in the estimates of consumption c and labor n, this correction is operated in the simulated values of these two variables. For instance, for a specific calibrated value of τ, the simulated series for consumption can be obtained as the deterministic steady state value of that variable times its dynamic behavior in the presence of one or more shocks (given as a percentage change):

$$c_t(\tau > 0) = c^{SS}_{\tau>0}(1 + \hat{c}_{\tau>0})$$

Where SS stands for Steady State and $\hat{c}$ represents the percentage change in consumption due to the stochastic innovations in z_t or ξ_t.

As can be observed in Table 1.10 in the Appendix, steady state utility levels, and consequentially, expected utility levels are higher when the tax advantage is present in the model ($\tau > 0$), rather than when it is absent. Therefore, in order to implement the correction I consider the model simulated without the distortion (i.e. the tax benefit) as the steady state starting point for all the simulated models, even for those where the calibrated value of τ was positive (i.e. $\tau = 0.35$).

The correction consists in multiplying the simulated series by the ratio of the steady state value of consumption when $\tau = 0$ over the steady state value of consumption when $\tau > 0$, that is:

$$c_t(\tau > 0)_{Corrected_{SS}} = c^{SS}_{\tau>0}(1 + \hat{c}_{\tau>0})\frac{c^{SS}_{\tau=0}}{c^{SS}_{\tau>0}} = c^{SS}_{\tau=0}(1 + \hat{c}_{\tau>0}) \tag{1.71}$$

And then we also apply this correction to labor:

$$n_t(\tau > 0)_{Corrected_{SS}} = n^{SS}_{\tau>0}(1 + \hat{n}_{\tau>0})\frac{n^{SS}_{\tau=0}}{n^{SS}_{\tau>0}} = n^{SS}_{\tau=0}(1 + \hat{n}_{\tau>0}) \tag{1.72}$$

With this correction, the simulated series for consumption and labor for all models departure from the same steady state levels, and consequentially the steady state utility levels will also be corrected as well:

$$U^i = E_0 \sum_{t=0}^{\infty} \beta^t ln(c_t(\tau > 0)^i_{Corrected_{SS}}) + \alpha ln(1 - n_t(\tau > 0)^i_{Corrected_{SS}}) \tag{1.73}$$

For $i = F, P, FB, BS$ and also for WS. Table 1.11 in Section 1.7.7 of the Appendix reports the estimates for the expected utilities of the 16 models corrected in the state state for the value of τ.

Finally, after obtaining the estimates for the corrected expected utilities, it is possible to compute the welfare costs of each shock, always taking into account that the ultimate purpose of these exercises is to isolate the welfare cost associated exclusively with the financial shocks.

1.6.2 Welfare Cost Calculation

As it was previously mentioned, the welfare cost of the financial shock is defined as the percentage change in the consumption of an agent in the economy with the shock necessary to achieve the same level of utility attained by an agent in the First Best equilibrium economy. In order to find a measure to quantify that welfare cost, let's consider again condition (1.67) and solve it in relation to λ:

$$E_0 \sum_{t=0}^{N} \beta^t ln(c_t^{FB}) + \alpha ln(1 - n_t^{FB}) = E_0 \sum_{t=0}^{N} \beta^t ln((1+\lambda)c_t^i) + \alpha ln(1 - n_t^i) \Leftrightarrow \tag{1.74}$$

$$\Leftrightarrow \lambda = e^{\left(\frac{1-\beta}{1-\beta^N}\right)(U^{FB} - U^i)} - 1 \tag{1.75}$$

For $i = F, P, BS$, and where $U^{FB} = E_0 \sum_{t=0}^{N} \beta^t ln(c_t^{FB}) + \alpha ln(1 - n_t^{FB})$.

Asymptotically, as $N \to \infty$ we have:

$$\lambda = e^{(1-\beta)(U^{FB}-U^{i})} - 1 \tag{1.76}$$

for $i = F, P, FB, BS$.

When we want to define the welfare effects in relation to the frictionless economy (WS) instead of using the First Best economy (FB) as the benchmark, then the condition that define the welfare measure becomes:

$$\lambda = e^{(1-\beta)(U^{WS}-U^{i})} - 1 \tag{1.77}$$

For $i = F, P, BS$, and where $U^{WS} = \frac{ln(c_t^{WS})+\alpha ln(1-n_t^{WS})}{1-\beta}$, since it is a deterministic steady state expected utility.

1.6.3 Results

Table 1.3 resume the main findings of the welfare cost analysis, computed using the procedure described in the previous section. This table contains the welfare costs of the financial and the productivity shocks, measured in percentage of the consumption of an agent of the frictionless economy without shocks and of the model with both shocks simultaneously active. This table reports the welfare costs results distinguishing the four possible cases regarding the calibration of the two main frictions present in this model: the tax subsidy τ and the equity payout cost parameter κ. Therefore, the findings reported refer to a comparison between two economies that will only differ in the type of shock but not in the type of friction that affects those economies. In that respect, those economies are equivalent, and therefore, the welfare costs reported are directly comparable within the same type of calibrated economy. Table 1.4 reports the differences in the magnitude of the welfare costs of the financial shock versus the productivity shock, measured as the ratio of the financial shock welfare cost over the productivity shock welfare shock, in relation of the frictionless economy (first row) and to the economy with both shocks (second row), also considering the four different combinations of parameterization for τ and κ.

Table 1.3: **Welfare Effects (in % of the consumption of an agent in the economy affected by the shock)**

	Financial Frictions							
	$\tau = 0.35$ $\kappa = 0.146$		$\tau = 0$ $\kappa = 0.146$		$\tau = 0.35$ $\kappa = 0$		$\tau = 0$ $\kappa = 0$	
MODEL	**F**	**P**	**F**	**P**	**F**	**P**	**F**	**P**
Without Shocks	0.0245	0.0013	0.0264	0.0014	0.0015	-0.0053	0	-0.0064
With Both Shocks	-0.0016	-0.0248	-0.0018	-0.0268	0.0054	-0.0013	0.0066	-0.0002

F - Model with only the Financial Shock; **P** - Model with only the Productivity Shock; τ - represents the tax benefit; κ - represents the equity payout cost

Table 1.4: **Relations Between the Welfare Effects of the Different Simulated Models (in % of consumption)**

Frictions	$\tau = 0.35$ $\kappa = 0.146$	$\tau = 0$ $\kappa = 0.146$	$\tau = 0.35$ $\kappa = 0$	$\tau = 0$ $\kappa = 0$
F/P(WS)	18.85	18.86	-0.283	0
F/P(BS)	0.065	0.067	-4.154	28.17

F/P(WS) - Ratio of the Welfare Cost of the Financial Shock over the Productivity Shock, computed in relation to the model Without Shocks; **F/P(BS)** - Ratio of the Welfare Cost of the Financial Shock over the Productivity Shock, computed in relation to the model with Both Shocks; τ - represents the tax benefit; κ - represents the equity payout cost parameter.

Table 1.5: **Welfare Effects of the Model with Both Shocks in comparison with the Economy Without Shocks (in % of consumption)**

Frictions	$\tau = 0.35$ $\kappa = 0.146$	$\tau = 0$ $\kappa = 0.146$	$\tau = 0.35$ $\kappa = 0$	$\tau = 0$ $\kappa = 0$
MODEL	**BS**	**BS**	**BS**	**BS**
WS	0.0261	0.0282	-0.0040	-0.0066

WS - Model without Shocks; **BS** - Model with Both Shocks; τ - represents the tax benefit; κ - represents the equity payout cost

At a first glance from Table 1.3, it is possible to observe that, in general, the estimated welfare effects according to this method are quite small in terms of magnitude, for almost all cases covered. On average, there is a welfare cost (instead of a welfare benefit) as a consequence of either the financial or the productivity shock, in comparison with a economy without shocks. However, there are some exceptions, that depend essentially in the presence or absence of the financial frictions, which I will later examine with more detail.

The most important findings reported in Table 1.3 refer to the estimates of the welfare effects computed in a simulation model in which both frictions are active (the tax benefit $\tau > 0$ as well as the equity payout cost $\kappa > 0$), and in comparison with the model without shocks, since these results confirm the expected scenario in which financial shocks have negative welfare consequences over the business cycle. The long-run welfare cost in the economy with only the financial shock relative to the economy without shocks represents 0.0245% of consumption, that is, each agent in the economy with only the financial shock would have to be compensated with an increase of 0.0245% on his consumption in order to be as well off as in the frictionless economy, when both $\tau > 0$ and $\kappa > 0$. The productivity shock also induces a welfare cost, but much smaller than in the case of the financial shock: an agent in a economy affected by the productivity shock would have to be compensated in 0.0013% of his consumption to be as well off as in the economy with no shocks.

However, in terms of magnitude, these welfare costs differ significantly between the two shocks, at least when the equity payout cost function is active (κ). For the economies with both frictions τ and κ, the welfare cost in the economy with only the financial shock (0.0245%) is nearly 18.85 times larger than the welfare cost in the economy with only productivity shock (0.0013%), when compared with the no-shock economy, as can be observed from Table 1.4. In fact, the welfare cost associated with the financial shock corresponds to approximately 93.87% of the welfare cost of the two shocks combined (which is of 0.0261%, as can be observed in Table 1.5) in relation to the economy without shocks, as it is possible to see by inspecting Table 1.3 and 1.5. One possible cause for this is the fact that the effects of the financial shock are largely amplified and propagated due to the presence of the tax advantage τ (which, as it was shown by Jermann and Quadrini (2012), assures that the enforcement constraint is always binding as long as $\tau > 0$ in the steady state, or as long as it is large enough when uncertainty is present), and due to the equity payout cost function, which, the higher the κ, the stronger will be the rigidity affecting the substitution between debt and equity, forcing the firms to cut employment n_t to maintain their production decisions unchanged and to keep satisfying the enforcement constraint.

But we cannot abstract from the fact that these welfare effects are computed from simulations of economies in which both frictions are present and where we are hitting those economies with only the financial and the productivity shocks. In other words, given that both the economy with the financial shock and the economy without shocks are simulated assuming that both frictions are active, the estimates presented in Table 1.3 only reflects the isolated welfare effect of the financial shock over the economy. The distortions in welfare that are due to the presence or absence of the frictions are controlled for, and hence this large difference in the welfare costs magnitude between the financial and the productivity shock can only be explained by the amplification and propagation effects of the financial shock, as it is defined and introduced in the model.

If we consider the welfare effects of each shock when compared to the economy with the two shocks simultaneously, the situation is precisely the reverse, as it would be expected: each agent in the economy with only the financial shock would have to reduce his consumption 0.0016% in order to attain the same utility level as in the two-shocks framework, while in the economy with only the productivity shock this reduction would have to be of 0.0248%. The fact these two values are negative naturally means that agents in the economy with both shocks are worse off than in the economies affected with only one shock. In this case, when comparing the economies with just one shock against the model with both, we can consider these estimates as welfare gains, instead of costs.

The rest of Table 1.3 gives us a clear insight of the influence of the financial frictions (τ and κ) in the welfare effects findings.

When we keep the equity payout cost active, i.e. $\kappa > 0$ and omit the tax benefit i.e. $\tau = 0$, the welfare situation is quite similar to the previous one, both in terms of direction and magnitude of the effects. In this case, the welfare cost of the financial shock corresponds to 0.0264% of consumption, while the welfare cost of the productivity shock is much lower, no more than 0.0014% of consumption. In comparison to the economy with both shocks, the situation is once again the opposite. It is possible to verify that two facts remain very close to the scenario where $\tau > 0$: the relation between the welfare costs associated with the two shocks and the weight associated with each of them in relation to the welfare cost imposed by the shocks in simultaneous. In this scenario the welfare cost of the financial shock is approximately 18.86 times higher than the welfare costs imposed by the productivity shock, when they are compared against the deterministic economy, and the situation reverses when the welfare effects are calculated in comparison with the two-shocks economy, as it is clear from Table 1.4. It is also possible to check that the welfare costs of the financial shock corresponds to approximately 93.62% of the welfare cost of the economy with both shocks (0.0282%) and, while the productivity shock only accounts for 4.9645% of that value.

The omission of the tax benefit does not bring dramatic changes to the welfare cost analysis, since, as an example, we can see that the importance of the welfare cost of the financial shock remains quite high (especially when compared to the productivity shock welfare cost), but only increased 0.0019 percentual points, which is a small increment in relation to the situation in which the tax benefit is active. The simulations for this scenario in which $\kappa > 0$ and $\tau = 0$ were conducted assuming that the enforcement constraint is always binding, even with uncertainty, although it is not possible to ensure, for $\tau = 0$, that this assumption is always true (at least outside the steady state). One possible explanation for this slight increase in the welfare cost is that without access to the tax-advantaged debt, the firm will not be so tempted to issue new debt in order to compensate the slow adjustment of the equity payout. With the reduction in the issuance of debt and since the adjustment of the equity payout is still costly, it is harder for the firm to finance capital investment, which reduces the capital stock and the productivity, and also decreases output. This contributes to reinforce the impact of the financial shock, which is mainly due to the presence of the equity payout cost, and which makes a change in ξ produce a larger movement in μ, tightening the enforcement constraint and leading to a larger amplification and propagation effects of the financial shock over the economy, also affecting negatively welfare.

However, when we analyse the scenarios in which $\kappa = 0$, i.e. the equity payout adjustment cost is absent, the results diverge considerably from the previous two cases. From Table 1.3, when $\kappa = 0$, the welfare effects estimates are not so intuitive as the precious findings, because although the estimates are positive for the financial shock when computed against the economy without shocks, we get negative results for the productivity shock. In other words, an agent in the economy hit only by the productivity shock would have to decrease his consumption in order to attain the same utility level as in a framework with no shocks, when the equity payout cost is absent (and firms are able to easily change their financial structure). This is true for the economy with only the productivity shock when compared to the no-shock framework whether the tax benefit is present in the model or not. One possible explanation for this has to do with the mechanism triggered by the introduction of the equity payout cost in the model: when $\kappa > 0$ and a positive productivity shock hits the economy, firms increase their investment and their capital stock to take advantage of higher productivity levels, which induces an amplification effect over output, consumption, wages and the interest rate, and consequentially benefits the welfare of the economy.

As for the financial shock, it produces a very small welfare cost (only 0.0015% of consumption) when the tax advantage is included in the model, but in the case where both financial frictions are excluded ($\kappa = 0$ and $\tau = 0$) the welfare effect is zero. This result is expected, since from Proposition 5 we know that when $\kappa = 0$ and $\tau = 0$, the model becomes a standard RBC with only the productivity shock.

Since in this case it is assumed that $\kappa = 0$ and $\tau = 0$ and we are comparing the economy with only the financial shock and another without shocks, we can conclude that these two frameworks are identical (the financial shock does not produce any disturbance in the economy, given these assumptions).

It is also important to take into account that the economy affected with only the productivity shock and with no financial frictions ($\tau = 0$ and $\kappa = 0$) represents the First Best solution of this framework, i.e., the economy from where it is not possible to move without decreasing the utility level of its agents. Since the standard RBC economy (with no frictions and only the technology shock) represents the First Best benchmark of this model, I also computed the welfare effects in relation to that setting, considering all the other possible scenarios of this model, regardless of the inclusion or omission of the financial frictions. The results are reported in Table 1.6. In general, in terms of magnitude of the welfare effects, the findings are very close to the ones reported in Table 1.3 and 1.5. Of course the results change slightly, but it is important to stress that in this case we are comparing economies with different assumptions about the inclusion/exclusion of the frictions, and that implies that the steady state equilibrium values imply different expected utility levels, although they are corrected according to the procedure described earlier in section 1.6.2. As we can observe, all the findings indicate that there is in fact a welfare cost that an agent in each one of the different economies simulated has to incur if he wants to be as well off as in the benchmark economy. The only exception seems to be the welfare cost of both shocks in an economy without the financial frictions (an agent in that economy would have to decrease his consumption 0.0002% in order to achieve the same utility level as in the First Best economy). Theoretically, this estimate should equal zero, since Proposition 6 states that when $\kappa = 0$ and $\tau = 0$ the financial shock has no real effects over the economy and therefore the model becomes the standard RBC framework with only the productivity shock. However, since the value of the estimate is quite low, it is almost negligible and the fact that is different from zero may result from approximations issues generated in the simulations of both models.

In terms of magnitude, the differences between the financial and productivity welfare costs are smaller than in the previous analysis, as can be observed in Table 1.7. When both financial frictions are active, the financial shock induces a welfare cost approximately 4 times higher than the one caused by the technology shock alone, when compared to the benchmark economy. Once again, this differences can be misleading, since we are comparing simulated models with different assumptions about the financial frictions, which affect the steady state levels of the variables.

Table 1.6: **Welfare Costs (in % of consumption) in relation to the Standard RBC Model** (only the productivity shock and no frictions)

Frictions	$\tau = 0.35$ $\kappa = 0.146$			$\tau = 0$ $\kappa = 0.146$		
MODEL	**F**	**P**	**BS**	**F**	**P**	**BS**
RBC	0.0307	0.0075	0.0323	0.0328	0.0078	0.0346
Frictions	$\tau = 0.35$ $\kappa = 0$			$\tau = 0$ $\kappa = 0$		
MODEL	**F**	**P**	**BS**	**F**	**P**	**BS**
RBC	0.0075	0.0007	0.0021	0.0064	**0.0000**	-0.0002

F - Model with only the Financial Shock; **P** - Model with only the Productivity Shock; **BS** - Model with Both Shocks; τ - represents the tax benefit; κ - represents the equity payout cost

Table 1.7: Relations Between the Welfare Costs of the Different Simulated Models (in % of consumption)

Frictions	$\tau = 0.35$ $\kappa = 0.146$	$\tau = 0$ $\kappa = 0.146$	$\tau = 0.35$ $\kappa = 0$	$\tau = 0$ $\kappa = 0$
F/P(RBC)	4.0933	4.2051	10.2282	–

F - Model with only the Financial Shock; **P** - Model with only the Productivity Shock; τ - represents the tax benefit; κ - represents the equity payout cost

1.7 Conclusion

Do financial shocks that affect firms' ability to borrow induce significant welfare costs to the economy? The research developed in this paper suggest that they do. Using the model developed by Jermann and Quadrini (2012), which incorporates some financial frictions formalized as a tax benefit and an equity payout cost that try to mimic the financial flows associated with firms' decisions regarding their financial structure, i.e. debt and equity financing, I followed a strategy based on Lucas (1987), later updated by Lucas (2003) to measure and compute the welfare effects associated with a financial shock representing the tightening of firms' financing conditions (specially during recessions) and a standard productivity shock.

I have investigated the welfare effects for sixteen different scenarios of this model, assuming different calibrations for the parameters that represent the financial frictions (τ and κ) and assuming different combinations of the inclusion and/or exclusion of the financial and the productivity shock. Those welfare effects were computed in relation to an economy without frictions and without shocks and also in relation to the standard RBC model with only the productivity shock and where the financial frictions are absent.

The findings suggest that the welfare costs caused only by the financial shock also small, but non negligible, specially when compared with the welfare costs of the productivity shock, since the former are approximately 18 times larger than the latter when computed in comparison with the frictionless model and approximately 4 times larger when compared with the RBC benchmark economy. These findings are also specially relevant for recession periods, since the research conducted by Jermann and Quadrini (2012) revealed that tight financial conditions have also played an important role in the recent 1990-91, 2001 ans 2008 financial crisis.

One possible extension of this analysis is to include heterogeneity at the firms and the households level in order to infer if the welfare costs differ significantly between different segments of the population, and also include a more defined role for the lender (represented by financial institutions that include commercial banks, investment banks, brokers, dealers and other specialists) by introducing a financial intermediary into the model.

Appendix

1.7.1 A.1 - Derivation of the Enforcement Constraint

The decision to default arises after the realization of revenues but before repaying the intra-period loan. The total amount of liabilities is $l_t + b_{t+1}/(1 + r_t)$, that is, the intra-period loan plus the new intertemporal debt. At this stage, in terms of liquidity, the firm holds the total value of production in period t, that is, $l_t = F(z_t, k_t, n_t)$.

As stated previously, the lender acquires the right to liquidate the firm's capital in case of default. It is assumed that at the moment of contracting the loan the liquidation value of physical capital k_{t+1} is uncertain: with probability ξ_t the lender will be able to recover the whole value k_{t+1}, but with probability $1 - \xi_t$ the lender is unable to recover anything. The liquidation value cannot be observed before the actual default by neither the parts. It is also assumed that the firm has all the bargaining power in the renegotiation process and the lender only gets the threat value. Therefore, in order to derive the renegotiation outcome two separate cases must be considered.

1. **Liquidation value is k_{t+1}**:

   Since the lender can expropriate the whole capital, the firm has to make a payment that leaves the lender indifferent between liquidation and keeping the firm in operation. The amount of payment that the firm needs to do in order to satisfy this condition is $k_{t+1} - b_{t+1}/(1+r_t)$, and must promise to pay b_{t+1} at the beginning of the next period, when the intertemporal debt is due. Therefore, the ex-post value of defaulting is

   $$l_t + Em_{t+1}V_{t+1} - k_{t+1} + \frac{b_{t+1}}{1+r_t}.$$

   Where V_{t+1} is the cum-divident market value of the firm.

2. **Liquidation value is zero**:

   If the liquidation value is zero, liquidation is clearly not the best option for the lender. Instead, the best option is to wait to the next period when b_{t+1} is due. In the current period the lender gets no payments and the firm retains the liquidity $l_t = F(z_t, k_t, n_t)$. Hence, the ex-post default value is

   $$l_t + Em_{t+1}V_{t+1}$$

   When the debt is contracted, the expected liquidation value is

   $$l_t + Em_{t+1}V_{t+1} - \xi_t\left(k_{t+1} - \frac{b_{t+1}}{1+r_t}\right).$$

   Enforcement requires that the value of not defaulting is not smaller than the expected value of defaulting, that is,

   $$Em_{t+1}V_{t+1} \geq l_t + Em_{t+1}V_{t+1} - \xi_t\left(k_{t+1} - \frac{b_{t+1}}{1+r_t}\right),$$

   which can be re-arranged as the enforcement constraint that the firm has to face:

   $$\xi_t\left(k_{t+1} - \frac{b_{t+1}}{1+r_t}\right) \geq l_t$$

1.7.2 A.2 - The First Best Solution in the Deterministic Steady State

Let's denote by $\bar{X}$ the value of variable X at the non-stochastic (deterministic) steady state. At the non-stochastic steady state, $z_t = \bar{z}$. From equation (15) we get:

$$\frac{\bar{k}}{\bar{n}} = \left(\frac{\beta\theta\bar{z}}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\theta}} \tag{1.78}$$

From condition (16) we get:

$$\frac{\bar{k}}{\bar{n}} = \left(\frac{\alpha\bar{c}}{(1-\bar{n})(1-\theta)}\right)^{\frac{1}{\theta}} \tag{1.79}$$

Using the steady state equation of condition (14), and equations (18) and (19), we get an expression for the hours of work in steady state:

$$\bar{n} = \frac{(1-\theta)(1-\beta(1-\delta))}{(1-\theta+\alpha\bar{z})(1-\beta(1-\delta))-\alpha\delta\beta\theta\bar{z}} \tag{1.80}$$

From this condition, we can retrieve the expressions for capital, consumption and investment in the steady state:

$$\bar{k} = \left(\frac{\beta\theta\bar{z}}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\theta}} \left(\frac{(1-\theta)(1-\beta(1-\delta))}{(1-\theta+\alpha\bar{z})(1-\beta(1-\delta))-\alpha\delta\beta\theta\bar{z}}\right) \tag{1.81}$$

$$\bar{c} = \left(\frac{(1-\theta)(1-\beta(1-\delta))}{(1-\theta+\alpha\bar{z})(1-\beta(1-\delta))-\alpha\delta\beta\theta\bar{z}}\right) \left[\bar{z}\left(\frac{\beta\theta\bar{z}}{1-\beta(1-\delta)}\right)^{\frac{\theta}{1-\theta}} - \delta\left(\frac{\beta\theta\bar{z}}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\theta}}\right] \tag{1.82}$$

$$\bar{i} = \delta\left(\frac{\beta\theta\bar{z}}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\theta}} \left(\frac{(1-\theta)(1-\beta(1-\delta))}{(1-\theta+\alpha\bar{z})(1-\beta(1-\delta))-\alpha\delta\beta\theta\bar{z}}\right) \tag{1.83}$$

The steady state expressions for equity $\bar{d}$ and debt $\bar{b}$ can be obtained substituting the expressions for $\bar{n}$, $\bar{c}$ and $\bar{k}$ in the firm's budget constraint and in the enforcement constraint.

1.7.3 A.3 - Types of Simulated Models

Table 1.8: **Types of Simulated Models**

File	**Description**
model1	With both shocks (z_t and ξ_t) and both frictions ($\tau > 0$ and $\kappa > 0$)
model2	With both shocks (z_t and ξ_t) and $\tau = 0$ and $\kappa > 0$
model3	With both shocks (z_t and ξ_t) and $\tau > 0$ and $\kappa = 0$
model4	With both shocks (z_t and ξ_t) and no frictions ($\tau = 0$ and $\kappa = 0$)
model5	Without the financial shock ($\xi_t = 0$) and both frictions ($\tau > 0$ and $\kappa > 0$)
model6	Without the financial shock ($\xi_t = 0$) and no frictions ($\tau = 0$ and $\kappa = 0$)
model7	Without the financial shock ($\xi_t = 0$) and $\tau = 0$ and $\kappa > 0$
model8	Without the financial shock ($\xi_t = 0$) and $\tau > 0$ and $\kappa = 0$
model9	Without the productivity shock ($z_t = 0$) and both frictions ($\tau > 0$ and $\kappa > 0$)
model10	Without the productivity shock ($z_t = 0$) and no frictions ($\tau = 0$ and $\kappa = 0$)
model11	Without the productivity shock ($z_t = 0$) and $\tau = 0$ and $\kappa > 0$
model12	Without the productivity shock ($z_t = 0$) and $\tau > 0$ and $\kappa = 0$
model13	Without shocks ($z_t = \xi_t = 0$) and both frictions ($\tau > 0$ and $\kappa > 0$)
model14	Without shocks ($z_t = \xi_t = 0$) and $\tau = 0$ and $\kappa > 0$
model15	Without shocks ($z_t = \xi_t = 0$) and $\tau > 0$ and $\kappa = 0$
model16	Without shocks ($z_t = \xi_t = 0$) and no frictions ($\tau = 0$ and $\kappa = 0$)

1.7.4 A.4 - Welfare Cost Measure Derivation

In order to obtain the condition that determines the measure for the welfare costs of shocks in this model we use condition (74) and solve in relation to λ:

$$E_0 \sum_{t=0}^{N} \beta^t ln(c_t^{FB}) + \alpha ln(1 - n_t^{FB}) = E_0 \sum_{t=0}^{N} \beta^t ln((1+\lambda)c_t^i) + \alpha ln(1 - n_t^i) \Leftrightarrow \quad (1.84)$$

$$U^{FB} = E_0 \sum_{t=0}^{N} \beta^t ln((1+\lambda)c_t^i) + \alpha ln(1-n_t^i) \Leftrightarrow \tag{1.85}$$

$$U^{FB} = E_0 \sum_{t=0}^{N} \beta^t ln(1+\lambda) + E_0 \sum_{t=0}^{N} \beta^t (ln(c_t^i) + \alpha ln(1-n_t^i)) \Leftrightarrow \tag{1.86}$$

$$U^{FB} = \left(\frac{1-\beta^N}{1-\beta} \right) ln(1+\lambda) + U^i \Leftrightarrow \tag{1.87}$$

$$\lambda = e^{\left(\frac{1-\beta}{1-\beta^N}\right)(U^{FB}-U^i)} - 1 \tag{1.88}$$

for $i = F, P, BS$

When $N \to \infty$ we have:

$$\lambda = e^{(1-\beta)(U^{FB}-U^i)} - 1 \tag{1.89}$$

for $i = F, P, BS$.

1.7.5 A.5 - Steady State Results for different calibrations of τ

Table 1.9: **The Effect of the Tax Subsidy (τ) in the Steady State**

τ	**SS Period Utility**	**Expected Utility**	c^{SS}	n^{SS}	k^{SS}	b^{SS}	w^{SS}	d^{SS}	y^{SS}	dis^{SS}	R^{SS}	μ^{SS}
0	-0.913486	-52.2217	0.78702	0.30083	8.37959	2.32161	2.12005	0.10863	0.9965145	0.9825	1.01781	0.00000153
0.1	-0.904527	-52.2192	0.79314	0.30037	8.79302	2.63992	2.13514	0.11015	1.01296	0.982	1.01603	0.01072824
0.2	-0.89531	-52.2193	0.79985	0.30008	9.25907	2.99990	2.15229	0.11184	1.03133	0.9825	1.01425	0.02148352
0.35	-0.881045	-52.2195	0.81117	0.30000	10.07974	3.63684	2.18253	0.11478	1.06317	0.9825	1.01158	0.03766313
0.4	-0.876192	-52.2197	0.81532	0.30009	10.39236	3.88049	2.19397	0.11589	1.07513	0.9825	1.01069	0.04306763
0.5	-0.866381	-52.2201	0.82424	0.30049	11.08893	4.42543	2.21923	0.11831	1.10147	0.9825	1.00891	0.05389145
0.6	-0.856512	-52.2205	0.83406	0.30123	11.89910	5.06263	2.24803	0.12108	1.13156	0.9825	1.00712	0.06473193
0.7	-0.846721	-52.2214	0.84489	0.30238	12.85136	5.81603	2.28099	0.12426	1.16620	0.9825	1.00534	0.07558454
0.8	-0.837225	-52.2230	0.85683	0.30406	13.98423	6.71816	2.31881	0.12793	1.20649	0.9825	1.00356	0.08644333
0.9	-0.828354	-52.2277	0.869999	0.30641	15.35098	7.81426	2.36243	0.13223	1.25385	0.9825	1.00178	0.09730007

1.7.6 A.6 - Expected Utilities Not Corrected in the Steady State for different τ values

Table 1.10: **Expected Utilities Not Corrected in the Steady State for different τ values**

File	Steady State Utility	Expected Utility (100 simulations) (N=100000 periods)	Expected Utility (120 simulations) (N=100000 periods	Expected Utility (1 simulation) (N=7000000 periods)
model1	-0.881045	-50.3609	-50.3611	-50.3562
model2	-0.913486	-52.2151	-52.2152	-52.2101
model3	-0.881041	-50.3426	-50.3429	-50.3378
model4	-0.913455	-52.1951	-52.1953	-52.1898
model5	-0.881045	-50.3463	-50.3465	-50.3412
model6	-0.913455	-52.1934	-52.1939	-52.1898
model7	-0.913486	-52.1997	-52.1999	-52.1944
model8	-0.881041	-50.3420	-50.3422	-50.3368
model9	-0.881045	-50.3600	-50.3601	-50.3596
model10	-0.913485	-52.1992	-52.1992	-52.1992
model11	-0.913486	-52.2143	-52.2142	-52.2144
model12	-0.881041	-50.3461	-50.3461	-50.3459
model13	-0.881045	-50.3454	-50.3454	-50.3454
model14	-0.913486	-52.1992	-52.1992	-52.1992
model15	-0.881041	-50.3452	-50.3452	-50.3452
model16	-0.913485	-52.1992	-52.1992	-52.1992

Note: The differences between each type of model presented in this table are reported in Table 8 (Types of Simulated Models) of section A.3 of the Appendix.

1.7.7 A.7 - Expected Utilities Corrected in the Steady State for different τ values

Table 1.11: **Expected Utilities Corrected in the Steady State for different τ values**

File	**Steady State Utility**	**Expected Utility (100 simulations) (N=100000 periods)**	**Expected Utility (120 simulations) (N=100000 periods**
model0	-0.881045	-52.2137	-52.2139
model2	-0.913486	-52.2151	-52.2152
model3	-0.881041	-52.1965	-52.1966
model4	-0.913485	-52.1951	-52.1953
model5	-0.881045	-52.1995	-52.1997
model6	-0.913485	-52.1953	-52.1955
model7	-0.913486	-52.1997	-52.1999
model8	-0.881041	-52.1957	-52.1959
model9	-0.881045	-52.2130	-52.2130
model10	-0.913485	-52.1991	-52.1991
model11	-0.913486	-52.2143	-52.2142
model12	-0.881041	-52.1997	-52.1997
model13	-0.881045	-52.1990	-52.1990
model14	-0.913486	-52.1991	-52.1991
model15	-0.881041	-52.1989	-52.1989
model16	-0.913485	-52.1991	-52.1991

Note: The differences between each type of model presented in this table are reported in Table 8 (Types of Simulated Models) of section A.3 of the Appendix

Chapter 2

The Importance of Financial Shocks for the Predictability of Recessions

2.1 Introduction

The importance of financial shocks as a source of serious economic downturns has been increasingly accepted in recent years by several authors, and currently, this acknowledgement is indisputable. Although the literature related with financial and banking shocks has expanded significantly (Bernanke et al. (1999), Guerron-Quintana (2009), Christiano et al. (2010), He and Krishnamurthy (2012), Kiyotaki and Moore (2012), Krishnamurthy (2010), He et al. (2010), Mendoza and Smith (2006), Mendoza (2010), Kiyotaki and Moore (2012), Negro et al. (2017) are recent examples of that trend), and the number of theoretical models that aim to explain how those financial frictions affect the real side of the economy has proliferated at a steady pace since the 2008 financial crisis, not all of those frameworks are able to properly replicate the behavior of financial and real variables during those episodes, especially in terms of volatility and magnitude of the shocks.

The main objective of this paper is to compare the empirical time series of major macroeconomic (financial and real) aggregates against the simulated series obtained using the model proposed by Jermann and Quadrini (2012), in terms of persistence, propagation and amplitude effects caused by a financial and a productivity shock. The impulse response functions are also computed in relation to both shocks. The ultimate goal is to infer if this framework is able to replicate the behavior of those financial and real variables during and after events as the 2008 financial crisis as it is described in the data.

In order to characterize the general environment that surrounded the 2008 financial crisis, a short survey is also included in this paper, in which I describe the main causes, triggers and consequences of that recession, since its early beginnings with the burst of the housing bubble at the end of 2006, including the start of the collapse of the financial system at the end of 2007 with the subprime crisis, and until the later period of worldwide economic depression which is now denominated the Great Recession.

In this paper I analyse the empirical behavior of financial and real variables during and after the recent 2008 financial crisis. I establish a comparison between empirical evidence based on macroeconomic data available for the U.S. before, during and after the collapse of financial system in the U.S. in 2008, and the results provided by the simulation of the Jermann and Quadrini (2012) model for the same time period.

In order to achieve these goals, I start by providing a general overview and context of the financial crisis, through a brief description of the major events which characterize the "oficial" time interval [1] of this crisis settled by NBER.

The data collected includes U.S. quarterly data which covers the period between the first quarter of 1952 until the second quarter of 2014, and it is based upon the same database used by Jermann and Quadrini (2012) [2].

[1]The U.S. National Bureau of Economic Research (NBER) officially sets the beginning of the recession at the end of the fourth semester of 2007 and sets the end at the second quarter of 2009.

[2]The original sample ranged from the first quarter of 1952 until the second quarter of 2010, although the authors focused on the interval 1984.I-2010.II to perform the simulations of their model.

Since the Great Recession originally started in the U.S. with the subprime financial crisis, I focus essentially on U.S. data. Although the full sample ranges between 1952.I-2014.II, I focus my analysis essentially on the period between 1984.I - 2014.II, because it includes the time range originally chosen by Jermann and Quadrini (2012) (1984.I - 2010.II) to calibrate their model and perform simulations. One of the major exercises of this paper is precisely to quantify and evaluate the main changes in the calibration of the model caused by the inclusion of this new set of observations in the original sample, and also study the performance of the model while replicating the data, specially in terms of the response of the major aggregates to exogenous shocks.

The data relative to the period between the third quarter of 2010 and the second quarter of 2014 is also heavily scrutinized, since this corresponds to the time range not covered by the original Jermann and Quadrini (2012) sample. This post financial crisis period coincides with the peak of the so called Great Recession, in which the negative effects of the initial financial crisis had already led to a worldwide slowdown that affected all the sectors of the economy.

All time series (financial and real) included in this study are seasonally adjusted. The major empirical sources are the Flow of Funds Accounts of the Federal Reserve Board, the Federal Reserve Economic Data (FRED) database of the Federal Reserve Bank of Saint Louis, the Bureau of Labor Statistics, and the National Income and Product Accounts (NIPA) database.

The methodology adopted in this paper is described as follows: first, the database originally used by Jermann and Quadrini (2012) is updated and extended until the second quarter of 2014 (1952.I-2014-II); then series for capital productivity, debt stock, equity payout, debt repurchase and total factor productivity are computed, the along with the most important statistical moments of each series (mean, variance, standard deviation, correlations, etc). Then I simulate the Jermann and Quadrini (2012) model and compute statistical moments and IRF's for the main macroeconomic variables. Finally, I compare the results.

This paper is organized as follows: in Section 2, a brief discussion of the literature related with the 2008 financial crisis is presented, as well as a brief description of some of the subsequent theoretical macroeconomic models that were developed to explain the shocks that originated the Great Recession and the long lasting effects that those events triggered in the aftermath of the crisis in the global economy. Section 3 covers a concise survey describing the sequence of events that led to the 2008 financial crisis in the U.S. and characterizes the empirical behavior of financial and real variables during and after that period. Section 4 provides a general overview of the Jermann and Quadrini (2012) model, describes the data collected and the methodology adopted to update the dataset and construct the series for productivity and financial shocks, which are subsequently used to run the simulations of the model. Section 5 presents and describes the results of the simulations and discusses the main findings. Section 6 concludes.

2.2 Literature

Recently, this subject of the 2007 financial crisis has been object of several studies from many distinguished authors. In order to support and complement this empirical survey, I cover some papers from Hall (2010), citetBrunnermeier2009, Chari et al. (2008), Afonso et al. (2010), Adrian et al. (2012), Gorton and Metrick (2012) and Gorton (2010).

From these references, I highlight the contribution of Brunnermeier (2009), that presented an analysis aimed to "explain the economic mechanisms that caused losses in the mortgage market to amplify into such large dislocations and turmoil in the financial markets", and to describe "common economic threads that explain the plethora of market declines, liquidity dry-ups, defaults, and bailouts that occurred after the crisis broke in the summer 2007". This author provided a detailed description of financial markets, explained the securitization process since its origins and laid out a complete timeline of the main events of the 2008 financial crisis. Chari et al. (2008) exposed and debunked three claims about "the way the financial crisis affected the global economy and also presented three underappreciated facts about how the financial system intermediates funds between households and corporate businesses", and Gorton and Metrick (2012) conducted a survey that selected and summarized 16 documents, including academic papers and reports from regulatory and international agencies, that "covers the key facts and mechanisms in the build-up of risk, the panics in short-term-debt markets, the policy reactions, and the real effects of the financial crisis". Adrian et al. (2012) studies both aggregate and micro level data and highlights the shift in the composition of credit between loans and bonds, and construct a model of direct and intermediate credit to capture the key stylized facts that characterize the financial crisis period of 2007-2009.

In order to construct a credible empirical survey, it is important to provide a historical perspective and a theoretical background for understanding the recent financial crisis and its consequences. In Section 2, I cover some papers that provide important empirical results about financial crises, which constitute important clues to shed light on the 2008 financial crisis but also predict and prevent future recessions. For example, Reinhart and Rogoff (2010) use long historical time series on public debt, along with modern data on external debts and they show that strong increments in private debt are a "recurring antecedent to banking crises", and these banking crises, independently of its domestic or external origin, usually precede or accompany sovereign debt crises. Afonso et al. (2010) examines the impact of the financial crisis of 2008 on the federal funds market, specifically the bankruptcy of Lehman Brothers, and concludes that, after that event, "banks become more restrictive in which counterparties they lend to" and "amounts and spreads become more sensitive to borrower bank characteristics". Blanchard (2009) provides a discussion of how "could such a relatively limited and localized event (the subprime loan crisis in the United States) have effects of such magnitude on the world economy". The author answers that question by identifying the essential initial conditions which have shaped the crisis ("the underestimation of risk contained in newly issued assets; the opacity of the derived securities on the balance sheets of financial institutions; the connectedness between financial institutions, both within and across countries; and, finally, the high leverage of the financial system as a whole"); by identifying the two amplification mechanisms behind the crisis ("the sale of assets to satisfy liquidity runs by investors" and "the sale of assets to reestablish capital ratios"); by showing how the amplification mechanisms have played out in real time, moving from subprime to other assets, from institution to institution, and from the United States, first to Europe, and then to emerging countries; and by proposing a change in future regulation and policies in order for them to avoid a repeat of some of those initial conditions. Other authors such as Obstfeld and Rogoff (2009) argue that the global imbalances of the 2000s, specially in the U.S., caused by economic policies that included "the interaction among the Feds monetary stance, global real interest rates, credit market distortions, and financial innovation" created the toxic mix of conditions making the U.S. the epicenter of the global financial crisis.

Nowadays it is largely accepted by several authors such as Brunnermeier (2009) and Gorton and Metrick (2012) that the 2008 financial crisis has its origins in the transformation of the banking system in the last 30 years, from which we can highlight two major changes. First, the deregulation process of the banking system which led to the exponential increase in the demand for secondary market products such as derivative securities and for collateral; and second, the rise of the denominated shadow banking system, especially in the U.S.A., which settled its development largely in the securitization process of the banking system. This securitization process was developed mainly through the issuance of tranches of loans that came to be used as collateral in repo transactions, freeing other types of assets, mostly treasuries, for use as collateral for derivatives transactions and for use in settlement systems.

2.3 A Brief Survey of the 2007-2009 Financial Crisis

The main goal of this section is to answer two major questions regarding the 2008 financial crisis and the following global recession: which sequence of events and cumulative effects triggered the collapse of the U.S. financial system in 2008 and what were the main consequences of this financial failure over the worldwide economy until today?

In order to achieve these goals, this section provides a general overview and context of the financial crisis, through a brief description of the major events which characterize the "oficial" time interval [3] of this crisis settled by NBER.

In order to construct a credible empirical survey, it is also important to provide a historical perspective and a theoretical background for understanding the recent financial crisis and its consequences.

2.3.1 Overview and Context of the Crisis

Nowadays it is largely accepted by several authors such as Brunnermeier (2009), Gorton and Metrick (2012), and Gorton (2010) that the 2008 financial crisis has its roots in the transformation of the banking system in the last 30 years, from which we can highlight two major changes. First, the deregulation process of the banking system which led to the exponential increase in the demand for secondary market products such as derivative securities and for collateral; and second, the rise of the denominated shadow banking system, especially in the U.S., which settled its development largely in the securitization process of the banking system. This securitization process was developed mainly through the issuance of tranches of loans that came to be used as collateral in repo transactions, freeing other types of assets, mostly treasuries, for use as collateral for derivatives transactions and for use in settlement systems. Adrian and Shin (2010b) argued that the "financial crisis of 2007-09 highlighted the changing role of financial institutions and the growing importance of the shadow banking system, which grew out of the securitization of assets and the integration of banking with capital market developments. This trend was most pronounced in the United States, but it also had a profound influence on the global financial system as a whole."

[3] The U.S. National Bureau of Economic Research (NBER) officially sets the beginning of the recession at the end of the fourth semester of 2007 and sets the end at the second quarter of 2009.

This repo transactions refer to the sale and repurchase (”repo”) market, which is essentially a short-term market for firms, banks, and institutional investors that is based upon a form of banking that involves the short-term (mostly overnight) deposit of money on call, backed by collateral. A repurchase agreement (or ”repo”) is essentially a transaction in which one side of the transaction wants to borrow money and the other side wants to save money by ”depositing” in a safe asset. In this case, the borrowers are the banks and the lenders are the depositors, which are generally another firm such a bank, insurance company, pension fund, institutional investor, or hedge fund. Gorton and Metrick (2009) analyzed extensively the repo markets during the ”Panic of 2007-2008” by using a novel data set that includes credit spreads for hundreds of securitized bonds to trace the path of crisis from subprime-housing related assets into markets that had no connection to housing. They concluded that there was indeed a run the repo market and that ”the U.S. banking system was effectively insolvent for the first time since the Great Depression”.

When some institution deposits money, the collateral may involve a ”haircut” or margin, which is the percentage difference between the market value of the pledged collateral and the amount of funds lent. This haircut protects the depositor against the risk of borrower default by the bank. In other words, the size of the haircut reflects the credit risk of the borrower and the risk of the pledged collateral. Another important feature of repo is that the collateral can be rehypothecated, i.e., the collateral received by the depositor can be used in another transaction, that is, it can be used to collateralize a transaction with another party.

The securitization process and the housing boom registered during the 1990's and the 2000's [4] (the latter until the burst of the housing bubble in 2007) were two important factors behind the change in the risk sharing structure between economic agents. In fact, the government deregulation of financial institutions ultimately increased risk, by permitting such institutions to alter the composition of their asset portfolios towards more high-risk securities. This preference for riskier, short-term assets led to higher leverage ratios, and accelerated the spread of complex financial

[4] The Appendix proves a brief explanation of the development of the securitization process.

holding companies that replaced the long established separation between investment banks, commercial banks and insurance companies. This process of repackaging loans and passed them on to various other financial investors contributed to disperse or in some cases transfer the risk from one financial investor to another, shrinking the risk taken by banks. However, since banks increasingly financed their asset holdings with shorter maturity instruments during the pre-crisis period, their exposition to a liquidity shortage increased significantly.

The traditional roles attributed to these types of financial institutions has also changed and intertwined, redefining the main goals, functions and instruments of those institutions and their correct placement inside the financial and money markets worldwide. This is particularly important in macroeconomic models which include financial intermediaries, since the authors must identify and characterize exactly what is the role and the competences of the financial intermediaries included in the model and how those characteristics can affect the equilibrium of the model and the final results.

According to Gorton (2010), "the shadow banking system is, in fact, a real banking system", in which the traditional role of depositors is played by firms, banks or another institutional investors seeking a place to save cash in the short term, often money market funds and corporations, and the role of lenders is played essentially by financial firms such as banks, insurance companies, pension funds, institutional investors, or hedge funds) seeking cash to finance themselves.

The transfer of loans from the traditional banking system to the "shadow" banking system, through the securitization process, and the increasing banks' investment in shorter maturity assets, in contrast to long-term liabilities, created a maturity mismatch in banks' balance sheets, therefore exposing the banks to funding liquidity risk. These two trends in the banking system constituted two major sources of the lending and housing booms that were in the origin of the financial crisis. As a consequence, several major financial institutions have failed (Lehman Brothers, Bear Stearns, Northern Rock, AIG, etc), and several stock markets have fallen dramatically, especially in the week after the FED bailout plan was passed.

In order to better understand how all these financial concepts intertwined and combined each other, culminating in the deepest recession of the last decades, the next section presents a timeline of all the main events which preceded and occurred during the peak of the crisis, starting with the burst of the housing price bubble and the subprime mortgage crisis.

2.3.2 The Subprime Mortgage Crisis - A Timeline

First of all, it is important to stress that in the decade before the starting events that later led to the financial crisis in 2008, the U.S. economy was experiencing a low interest rate environment, both because of large capital inflows from abroad, especially from Asian countries, and because the Federal Reserve Bank had adopted a lax interest rate policy. The Federal Reserve feared a deflationary period after the bursting of the Internet bubble in 2001, and therefore did not counteract the building up of the housing bubble, which began since 2006.

The major event that is considered as the beginning of the financial crisis was "an increase in subprime mortgage defaults, which was first noted in February 2007", according to Brunnermeier (2009). Roughly at the same time, the phenomenon that was designated by some authors as the housing prices "bubble" burst out in the late 2006 and beginning of 2007. The housing price bubble is defined by the fact that house prices were rising fast and steadily more than fundamentals during the credit boom of the 1990's, although there is a large debate in the literature concerning the exact definition of a bubble [5]. U.S. home loan lenders and home-builders started to report successive losses, as can be observed in the S&P Case-Shiller 20-City Home Price Index in Figure 2.1. The bursting of the housing bubble forced banks to write down several hundred billion dollars in bad loans caused by mortgage delinquencies. The unexpected fall in the house prices also led to the reduction in construction activity. However, although housing construction is sufficiently large industry that

[5] In 2003, long before the burst of the bubble in 2007, Case and Shiller (2003) documented the house price increases, and presented two types of evidence that suggest that "fundamentals" cannot account for the price increases on their own.

this reduction would have shown up in the decline of overall GDP, this reduction is comparable in size to other recessions of the post WW2 era.

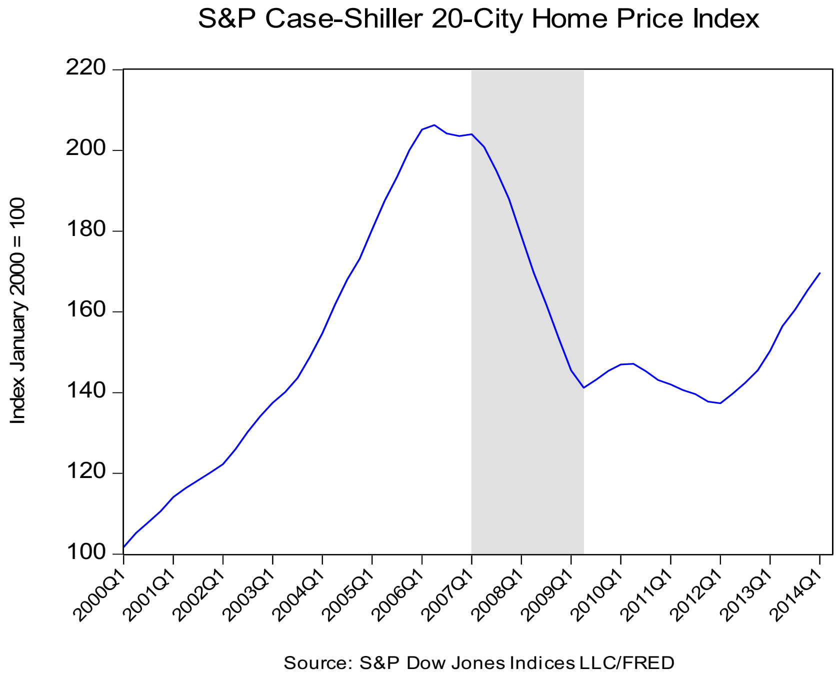


Figure 2.1: S&P Case-Shiller 20-City Home Price Index

After the gradual emergence of these events, by the end of July of 2007, financial and credit markets witnessed massive downgrades of mortgage-backed securities by the main rating agencies (Moody's, Standard & Poor's and Fitch), leading to a fast decline of the prices of mortgage-related products. During this period, we can take as examples two subprime mortgage underwriters (Ownit Mortgage Solutions and New Century Financial Corp.) that filed for bankruptcy, and the case of Bear Stearns, which in June of the same year began to show signs of distress since it had to inject nearly $3.2 billion in order to avoid the default of two of its own hedge funds. In Europe, the first victim of he subprime crisis was the German bank IKB, on July, since the bank proved unable to provide the promised credit line.

In August problems in mortgage and credit markets spill over into interbank markets, mainly through a run on repurchase or "repo" market, characterized by steep rise in price of haircuts and termination of repo market lending on collateral in form of asset-backed securities (ABS's) or asset-backed commercial paper (ABCP). This massive deleveraging lead quantitative hedge funds to suffer heavy losses. One of the most notorious examples of this phenomenon at this time was the run on the U.S. home loan lender Countrywide Financial Corp.

On August 9 the ECB injected 95 billion euros of liquidity into the interbank market, in a first attempt to avoid a systemic effect of the bank run over the traditional commercial banking system, and this action is then followed soon after by other central banks worldwide. The Federal Reserve Board approved a 50-basis-points reduction in the discount rate, "broadened the type of collateral that banks could post, and lengthened the lending horizon to 30 days", according to Gorton (2010) and Brunnermeier (2009). For the rest of this month, the participants of the interbank, money and commercial paper markets started to show several signs of distress and distrust in lending to each other, as the sudden increase in the interest on asset-backed commercial paper occurred during this period, as Figure 3.2 illustrates. This "liquidity crunch" was accompanied by a steep rise in the perceived default risks by banks, leading the LIBOR and other benchmark interest rates to rise temporarily. During this period, rating agencies systematically continued to downgrade several securitized products, such as conduits and structured investment vehicles.

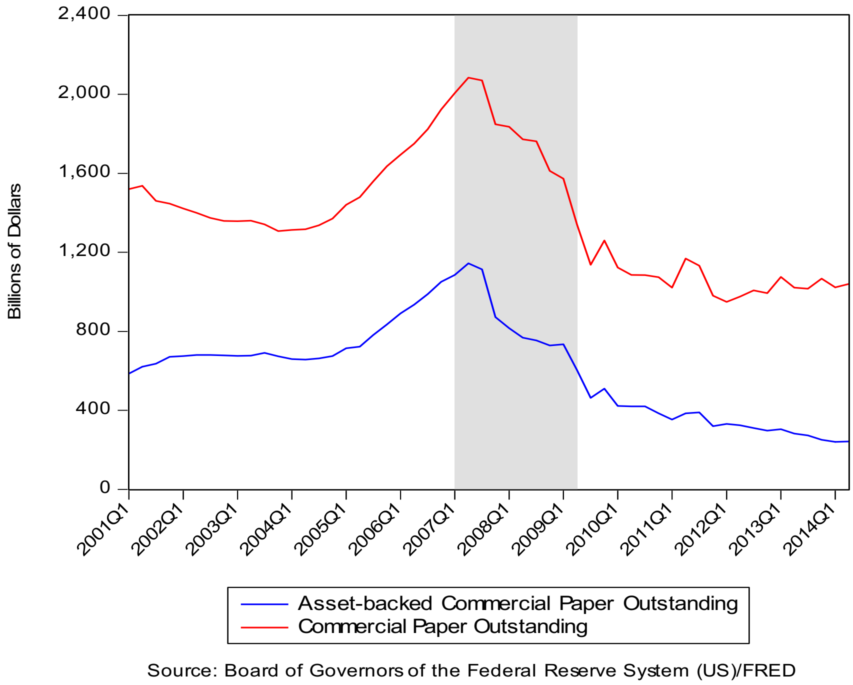


Figure 2.2: Asset Backed Commercial Paper Outstanding in the U.S.

On September 9 there was a run on U.K. bank Northern Rock, "the first in 150 years", according to Gorton (2010), and three days later the Fed decided to lower the federal funds rate by half a percentage point (50 basis point) and also the discount rate.

Between October and December several massive write-downs were registered by major financial firms, such as Citigroup, UBS and Morgan Stanley. In response, the Fed decided to reduce the federal funds rate by 0.25 percentage point on December 11 of the same year. Next day, in order to revive interbank lending, the Fed announced the creation of the Term Facility Auction (TAF), "through which commercial banks could bid anonymously for 28-day loans against a broad set of collateral, including various mortgage-backed securities", according to Brunnermeier (2009). Officially, the National Bureau of Economic Research has declared December of 2007 to be the business cycle peak of the subprime crisis until then.

Due to the fast increase in the number of write-offs and liquidations, on March of 2008 the Fed announced an increase of 40 billion dollars in the size of the TAF and also announced the creation of the Term Securities Lending Facility program in order to restore and promote liquidity into the interbank market. Through this unprecedented quantitative easing measure, the Fed expanded its securities by lending Treasury bonds against a range of eligible assets (see Gorton (2010) and Gorton and Metrick (2012)). According to Brunnermeier (2009), "some market participants might have (mistakenly) interpreted this move as a sign that the Fed knew some investment bank might be in difficulty. Naturally, they pointed to the smallest, most leveraged investment bank with large mortgage exposure: Bear Stearns." Since Bear Stearns was considered "too big to fail", i.e. too large and "interconnected" to allowed to fail suddenly, on March 16 JPMorgan Chase agreed to buy it, with Federal Assistance from the Fed of New York, which created the Primary Dealer Credit Facility, an overnight funding facility for investment banks in distress.

During those first months of 2008, several interest rate spreads associated with mortgage-backed securities (in relation to Treasury bonds) rose steadily, particularly those related to the government-sponsored enterprises Fannie Mae and Freddie Mac. Ultimately, after successive losses in the stock prices of these two institutions, the Federal government was forced to take over Fannie Mae and Freddie Mac on September of 2008.

Although it had survived the first liquidity crunch in March 2008 mainly because of the injection conceded by the Primary Dealer Credit Facility, Lehman Brothers' shares plunged on September 9. After all major banks declined the possibility to take over Lehman Brothers without a government guarantee, the investment bank, previously perceived as "too big to fail", was forced to file for bankruptcy on September 15, 2008.

After this, events started to unwind very quickly, and soon other large investment banks as Merrill Lynch and large international insurance companies as AIG (heavily linked to the credit securitized derivatives markets) began to declare huge liquidity shortages and huge losses on their stock prices, in the hope to be bailed out by the Fed. In fact, the Federal Reserve lent 85 billion dollars to AIG to avoid bankruptcy in exchange for an 80 percent equity stake.

Since Lehman Brothers had several affiliates and counterparties around the world, the effects of this bankruptcy soon spread into the global financial markets. Several money market funds, such as the Reserve Primary Fund, ”broke the buck” i.e. ”their share price dropped below $1”, causing a run on the money markets, according to Gorton and Metrick (2012) and to Brunnermeier (2009).

Fearing the serious consequences of a potential run on money market funds, on September 19 the U.S. Treasury announced a temporary $80 billion guarantee for brokers' money market funds, and the Federal Reserve announced the Asset-Backed Commercial Paper Money Market Mutual Funds Liquidity Facility.

Despite all the efforts conducted by the Fed to prevent a massive run on the credit market and on the money market funds, the deterioration of the financial stability continued to increase at an alarming rate. On September 25, Washington Mutual, one of the largest savings and loan funds in the U.S. was seized by the authorities (the Federal Deposit Insurance Corporation), after the majority of its ”costumers and fund managers withdrew funds electronically”, which is described by Brunnermeier (2009) as a ”silent bank run”.

By October, the U.S. overall stock market collapsed, ”losing about $8 trillion in the year after its peak in October 2007”, according to Brunnermeier (2009), and the financial crisis was spread to Europe and to the rest of the world, and its consequences were already spilling over the range of the financial markets, starting to affect the real side of the economy, as can be observed in the peak of 10.0% of the unemployment rate in October of 2009, in Figure 2.3.

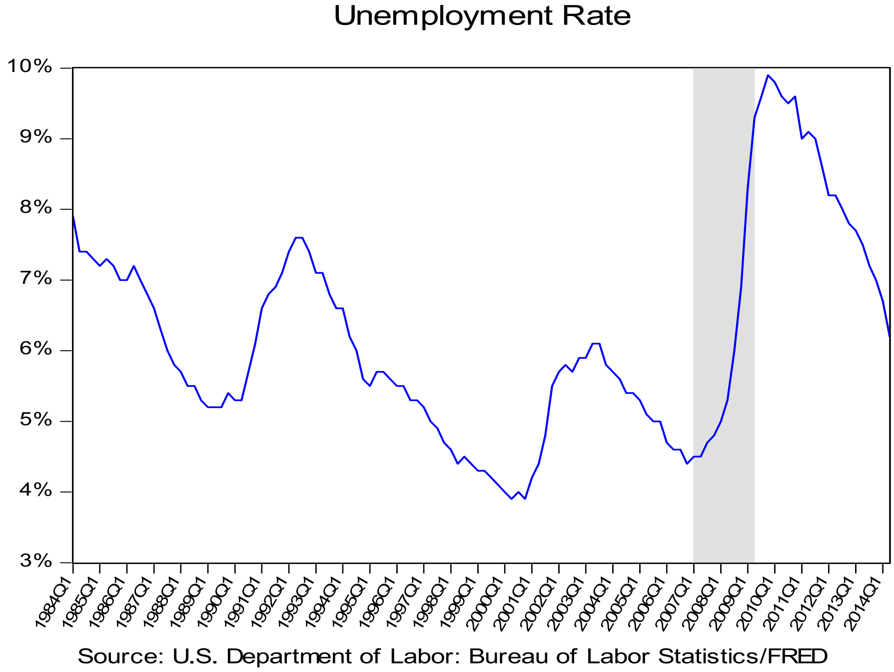


Figure 2.3: Unemployment Rate in the U.S.

From that point on, several central banks (U.S., England, China, Canada, Sweden, Switzerland, and the European Central Bank (ECB)) started to develop and implement a coordinated action in the financial markets, in order to help the world economy to overcome the crisis as soon as possible, which included measures such as the cut of major target interest rates, and the announcement of "unlimited provision of liquidity to U.S. dollar funds", according to Gorton and Metrick (2012). The Fed proposed and adopted a bailout plan (the Troubled Asset Relief Program (TARP)), that included "foreclosure-mitigation elements for homeowners, provisions to purchase troubled mortgage assets, and a coordinated forced recapitalization of banks." Various stock markets have fallen dramatically, especially in the week after the bailout plan implemented by the Fed was passed. Spreads on a variety of different types of loans over comparable U.S. Treasury securities have widened dramatically. However, since the number of distressed banks and the number of failures (see Figures 2.4 and 2.5) didn't cease to increase, in November 2008 the Fed announced the creation of several facilities that allowed the Fed to buy commercial paper and almost any type of asset-backed security and agency paper.

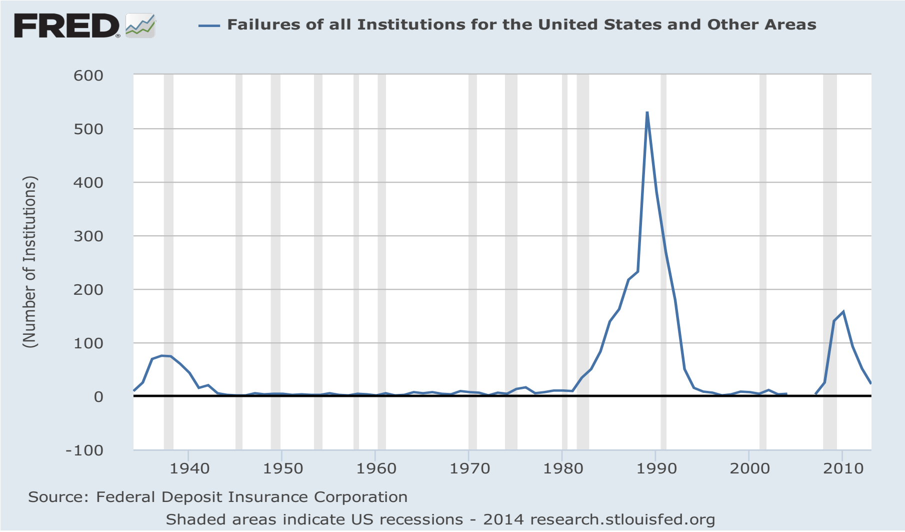


Figure 2.4: Number of Failures in the U.S.

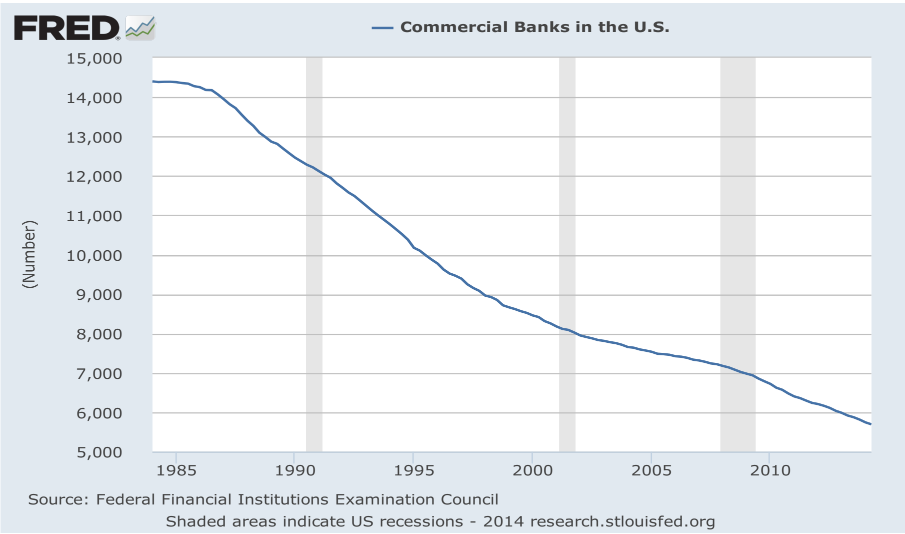


Figure 2.5: Number of Commercial Banks in the U.S.

This unprecedented adoption of extraordinary off-balance measures by the Fed (quantitative easing) triggered a massive increase in the size of the Fed's balance sheet, as can be observed in Figures 2.6, 2.7 and 2.8. The Fed started to buy unconventional securities like repurchase agreements, as can be observed in Figure 2.9. In fact, the Fed's total assets, which presented a very well behaved, constant path until the second quarter of 2008, more than doubled from about \$8.9 billion in that quarter to about \$2 trillion in December 2008. This unprecedented and huge absorption of more risky assets (such as commercial paper, asset backed securities and MBS's) by the Fed continued to increase irregularly after the end of 2008, but suffered a new jump in the third quarter of 2012, as can be observed in figure 2.8.

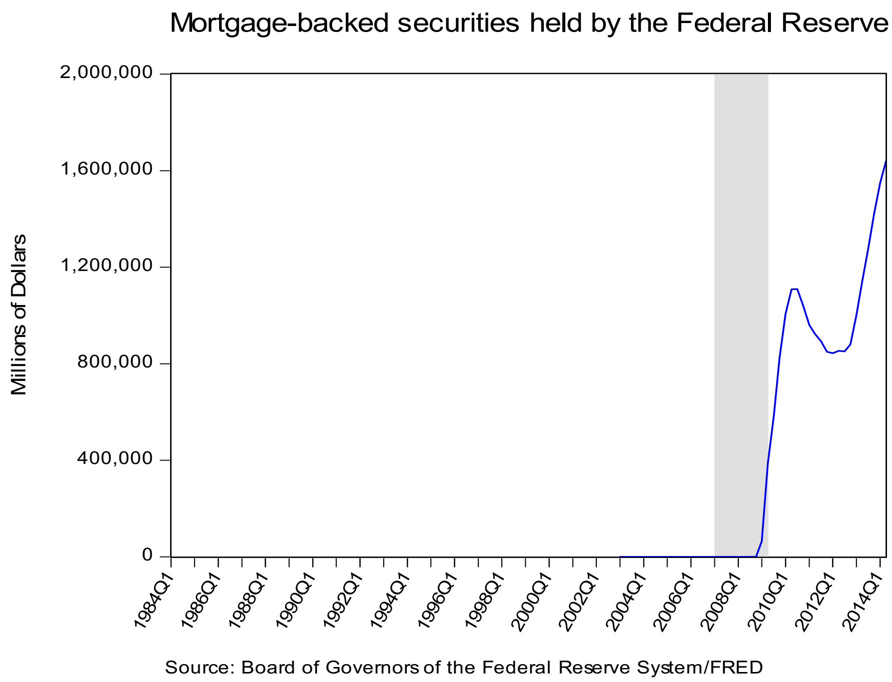


Figure 2.6: Mortgage-Backed Securities held by the Federal Reserve

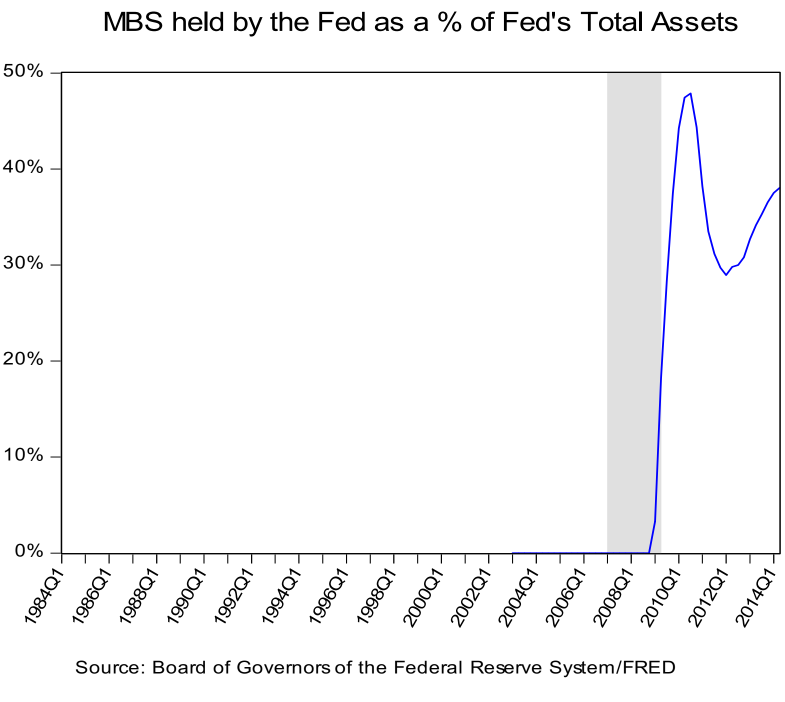


Figure 2.7: Mortgage-Backed Securities as a % of GDP

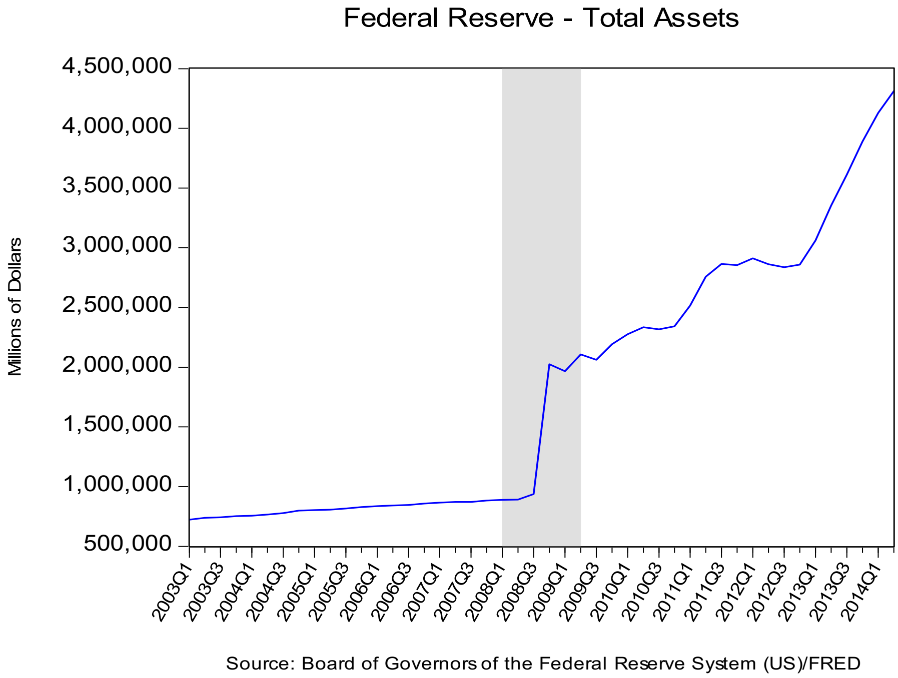


Figure 2.8: Federal Reserve Total Assets

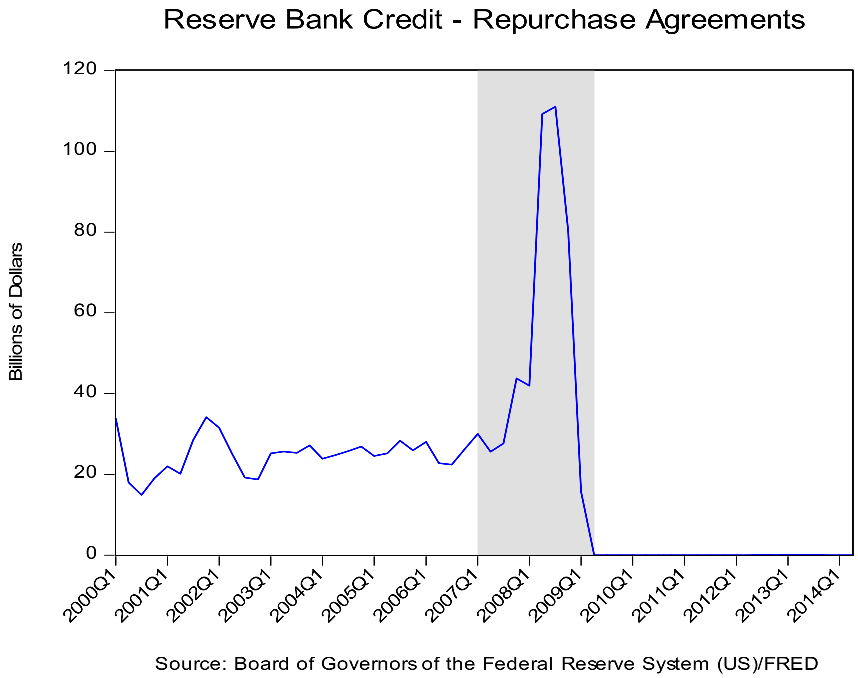


Figure 2.9: Repurchase Agreements held by the Federal Reserve

Overall, asset prices fell substancially during the 2008 financial crisis, especially the prices of non-subprime-related assets (see Figure 2.10). Financial firms could only deleverage in response to the increase in haircuts by selling assets. There was no suspension of convertibility (the banks refused to give back the cash to their depositors), therefore financial firms had to try to sell loans and mortgages. Commercial paper issued by financial institutions has declined, commercial paper issued by nonfinancial institutions is essentially unchanged during the financial crisis. Due to the quick succession of cases of financial institutions with shares collapses, imminent or effective bankruptcy, stock markets start to melt down since the last quarter of 2007, as can be observed in Figure 2.11.

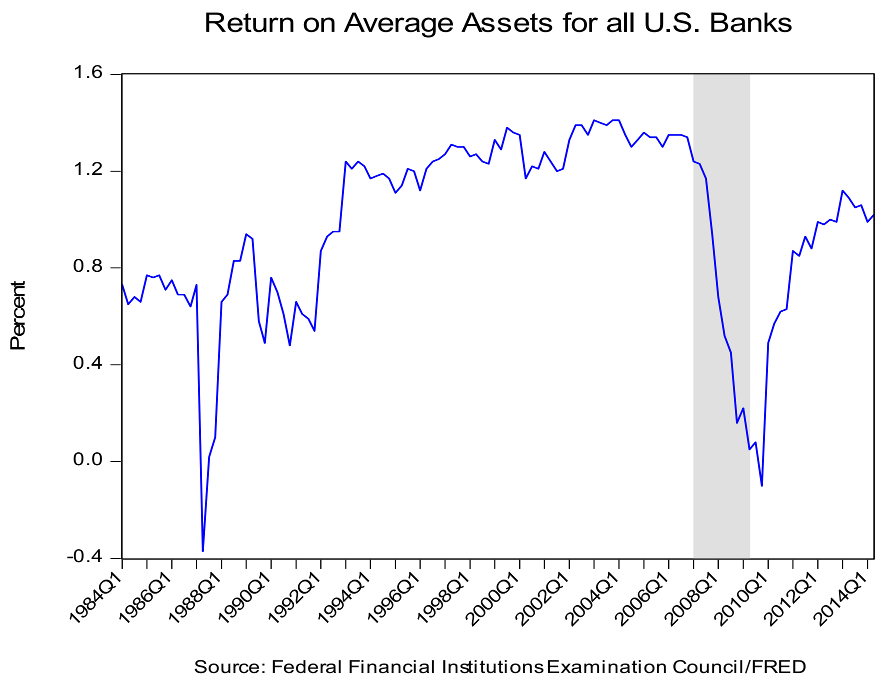


Figure 2.10: Return on Average Assets for all U.S. banks

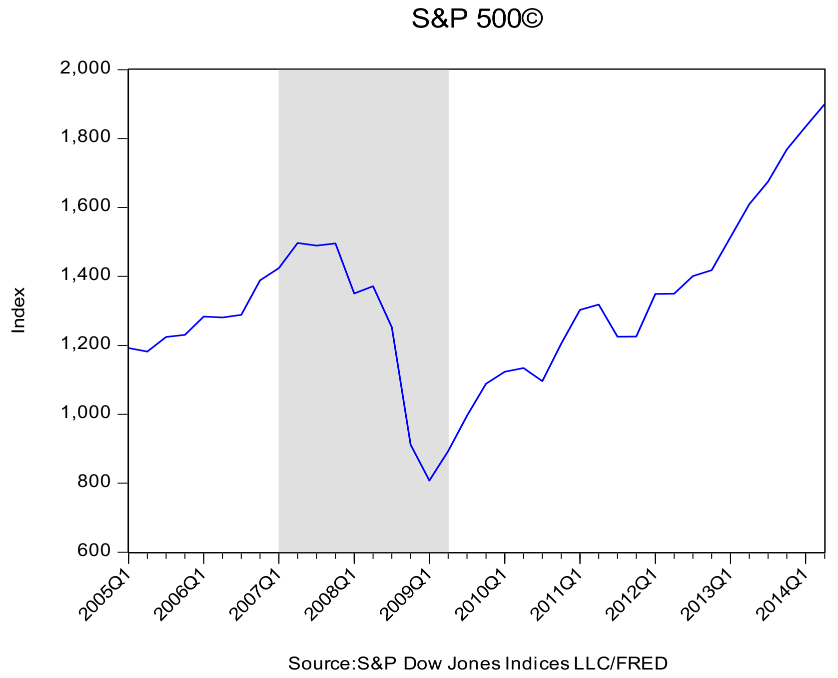


Figure 2.11: S&P 500 Dow Jones Index

However, contrary to what many studies may claim about the nature of the 2008 financial crisis, bank lending to nonfinancial and financial corporations and individuals has not declined during the financial crisis, but has actually increased, as Figures 2.12, 2.13, 2.14 and 2.15 illustrate. This claim was noticed, studied and documented by Chari et al. (2008), and these authors concluded that bank credit has not declined during the financial crisis but, on the contrary, actually appears to have risen relative to the trend that was preciously following. From Figure 2.12, it is possible to observe that the total amount of bank credit, defined as ”the aggregate amount of assets held by these banks excluding vault cash” by Chari et al. (2008), has actually increased until the last quarter of 2008, as well as the total amount of loans and leases that integrate total credit, as Figure 2.13 displays. If we decompose further total credit, its main components also followed an upwards path during the financial crisis, as Figure 2.13 illustrates. Consumer loans, commercial and industrial loans and real estate loans at all commercial banks have increased during the peak of the financial crisis, and only started cropping at the end of 2008 or at the beginning of 2009.

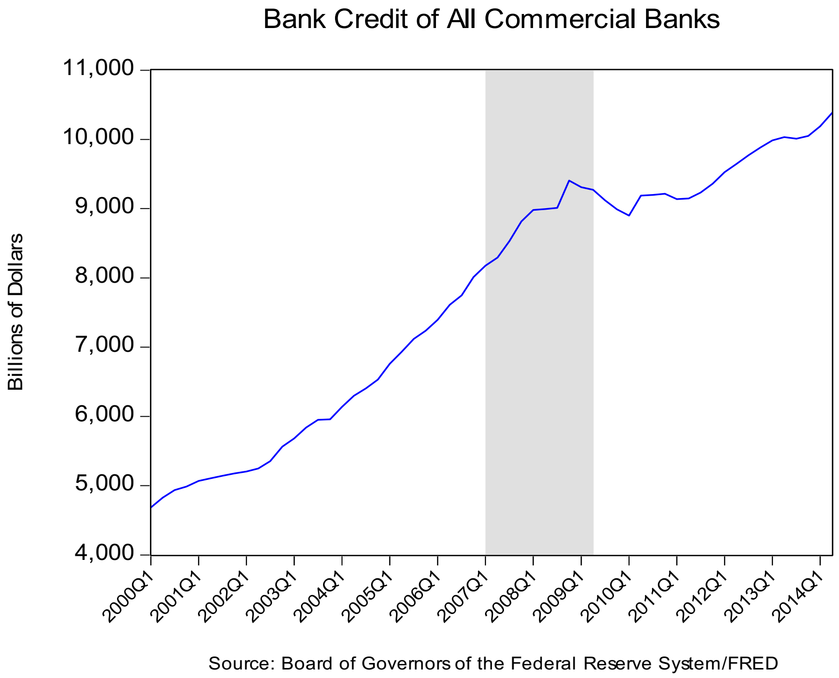


Figure 2.12: Bank Credit of All Commercial Banks

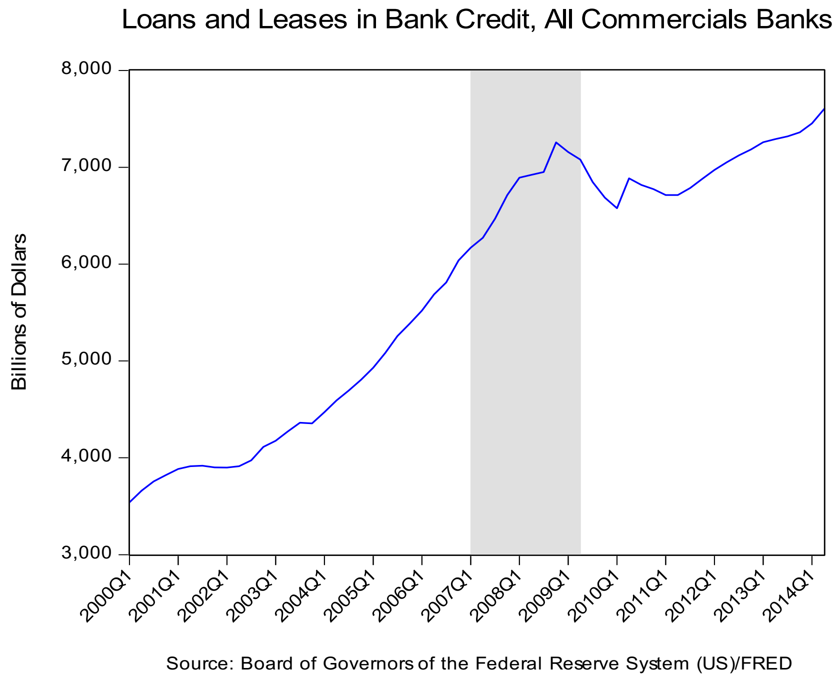
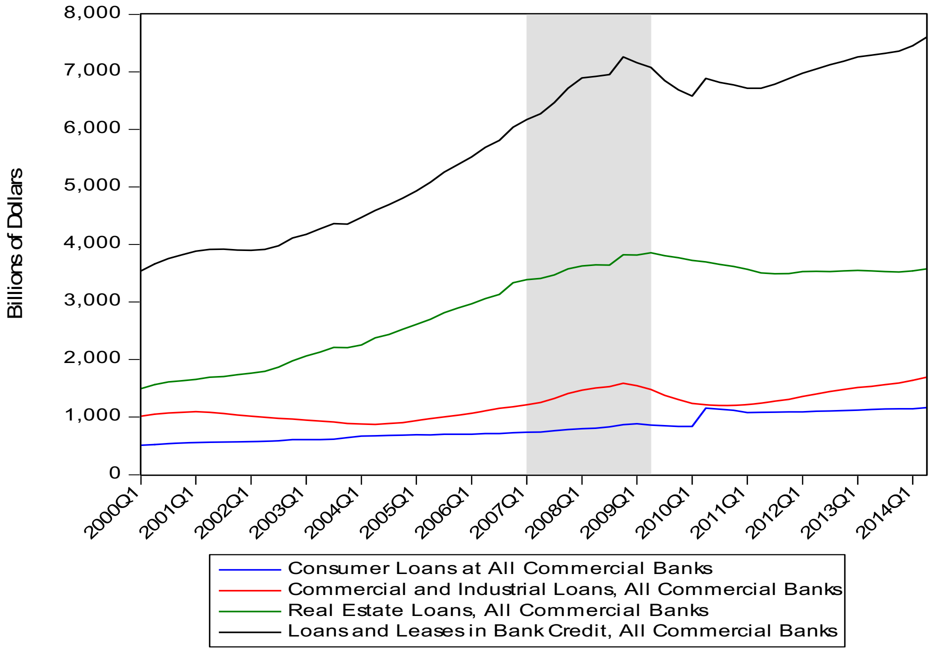
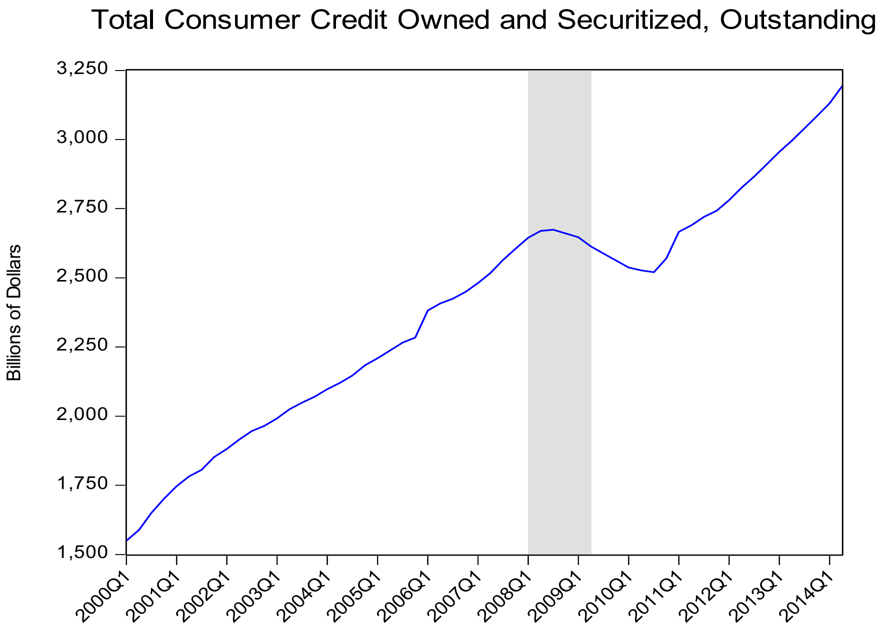


Figure 2.13: Loans and Leases in Bank Credit



Source: Board of Governors of the Federal Reserve System (US)/FRED

Figure 2.14: Evolution of different loans types



Source: Board of Governors of the Federal Reserve System (US)/FRED

Figure 2.15: Total Consumer Credit Owned and Securitized, Outstanding

2.3.3 Data Description

Nowadays it is generally accepted that the sequence of events that led to the financial crisis started during the first semester of 2007, and that the crisis reached its peak around the middle of 2008 [6]. However, when we analyze data from the last thirty years, it is very clear that the changes that occurred since then in the financial and banking sectors laid the foundations for the events that later triggered the 2007 crash of the financial system, leading up to one of the major recessions since the 1930's Great Depression.

In the empirical analysis which follows, I used quarterly data originally collected from Jermann and Quadrini (2012), which covers the period 1984.I-2010.II. I collected most recent data from the same sources as Jermann and Quadrini (2012) to update their series until the second quarter of 2014. I also collected other variables (mostly financial) in order to describe a wider scenario of the economic and financial environment that surrounded the U.S. financial markets and the banking system before, during and after the financial crisis. Most data was collected from the Bureau of Economic Analysis of the U.S. Department of Commerce, the Bureau of Labor Statistics of the U.S. Department of Labor and the Board of Governors of the Federal Reserve System, all through FRED, a database from the Economic Research of the Federal Bank of St. Louis which comprises over 267,000 economic time series from 80 sources. In this sample most variables are in logs and linearly detrended over all the time periods.

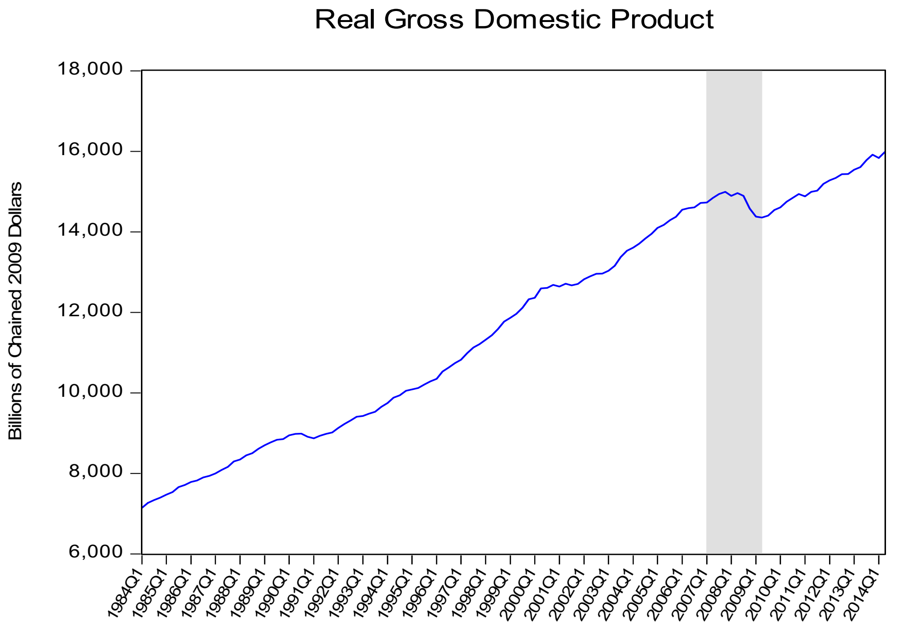
In real terms, since the beginning of the subprime mortgage crisis in 2007 until 2014, all the major macroeconomic variables have suffered heavy drops. Figure 2.16 displays quarterly data on the real Gross Domestic Product (GDP) in the U.S. from 1984 onward. As it is clear from this figure, since 1984 real GDP has not declined as deeply and sharply as during the current recession, which started in 2007 with the collapse of the U.S. financial markets. Although there was a downturn in

[6] The FRED database of the Economic Research Department of the Federal Reserve Bank of St. Louis officially settled that the financial crisis started on December 1, 2007 and ended in the second quarter of 2009, on June 1, 2009.

the beginning of the 1990's, the U.S. economy recovered relatively fast during the nineties due to the stock market boom [7], until the turn of the century. However, this boom was followed by a new slowdown caused by the 2001 dot-com bubble, which persisted in the following years, but was vastly surpassed after 2007. In fact, the 2008 financial crisis seems to mark a trend break in real GDP as well as in its main components.

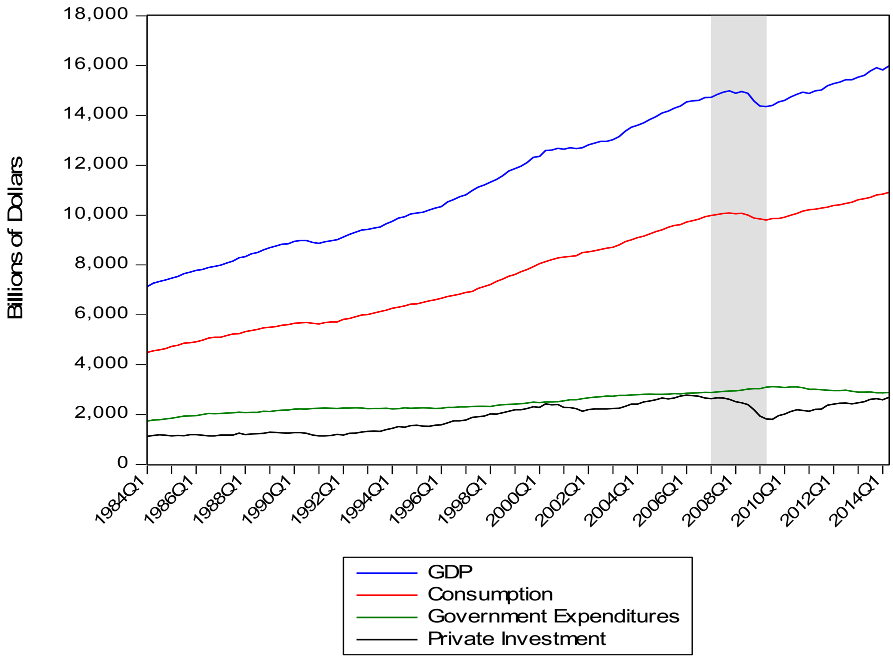
In order to illustrate this point, Figure 2.17 represents the evolution of real GDP along and its main components (except for net exports): private consumption, government expenditures and private investment. We can observe that all variables declined substantially during the financial crisis period, in comparison with the rest of the time interval considered, especially from the beginning of 2008, except for the government expenditures, possibly due to its countercyclical nature as an automatic stabilizer. Although the financial markets started to show signs of distress from the early beginning of 2007 on, the real side of the economy only started to collapse in the beginning of 2008. However, in order to take into account that the first signs of distress had its origins in the financial sector since January 2007, I highlight the period 2007:I - 2009:II in all figures as representing the 2007-2009 financial crisis.

[7] According to **?**



Source: U.S. Department of Commerce: Bureau of Economic Analysis/FRED

Figure 2.16: U.S. Real GDP



Source: U.S. Department of Commerce: Bureau of Economic Analysis/FRED

Figure 2.17: GDP and its main components

In Figure 2.18 it is possible to see with more detail the evolution of the cyclical behaviour of GDP and its components between the first quarter of 2000 until the second quarter of 2014. Like GDP, private consumption, government expenditures and private investment also registered a turning point during the "official" financial crisis period, around the second quarter of 2008, although with very different degrees of volatility (as expected, private investment is the most volatile variable). In general, all these variables are coincident with the economic activity and procyclical, although government expenditures presents a countercyclical behaviour, as it is expected if we consider its role as an automatic stabilizer. It is very clear from this figure that there is a time gap between the collapse of the financial system and the first signs of distress in the real side of the economy. Private investment, as it is expected according to the standard empirical literature, exhibits a leading behaviour, since it started its decline earlier than GDP, around the second half of 2007. This delay in the response of the real variables to a shock originated in the financial sector can be explained by several factors which resulted from direct authorities' intervention, such as the successive efforts of the Federal Reserve to contain the contagion effects of the financial shock into other sectors of the economy (or at least to minimize those spillovers). However, there are also more theoretical factors, which support that this delay can also be explained at the light of a pre-existent mechanism which linked the financial sector to the real side of the economy, conceding strong propagation and amplification effects of this particular shock.

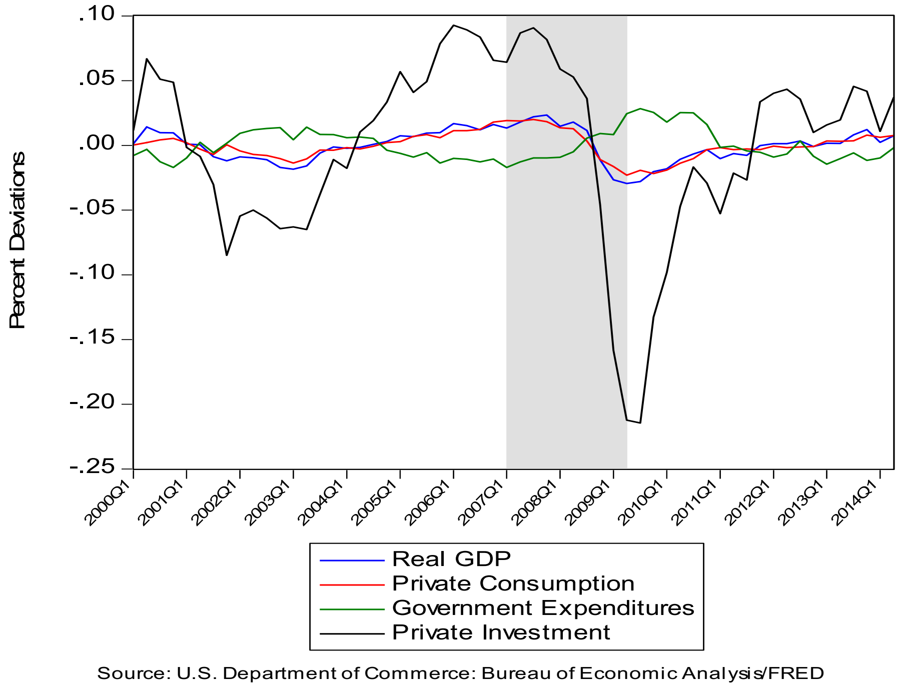


Figure 2.18: GDP and its main components (detrended)

In response to the breakdown in the production levels the unemployment rate suffered a huge and sudden increase. Figure 2.3, previously presented in section 3.2, displays data for the unemployment rate in the U.S. since the first quarter of 1984 until the second quarter of 2014. Although there was a small jump from approximately 4% to 6% from 2001 and 2004, due to the internet bubble, the economy recovered until the end of 2006. Then, matching with the beginning of the financial crisis, from 2007 on, the unemployment rate raised steeply until reach almost 10% in 2010. From the beginning of 2011 the unemployment rate started to decrease slowly, and is almost returning to the 6% 2004 levels. The economy is starting to recover, but the jump in the unemployment rate was the highest in the last thirty years.

The negative spillovers of the financial crisis also affected the production factors, labor and capital. Figure 2.19 displays an index of aggregate weekly hours of all employees, and from this graph it is clear that its volatility has increased substantially in the last fifteen years, after presenting a relatively increasing trend since 1984. Although this variable has suffered a significant decrease since the beginning of the 2000's, its breakdown deepened significantly a year after the breakthrough of the financial crisis in 2007. However, in the beginning of 2010 the number of hours started to recover slowly, although it has not yet reached the pre-crisis levels.

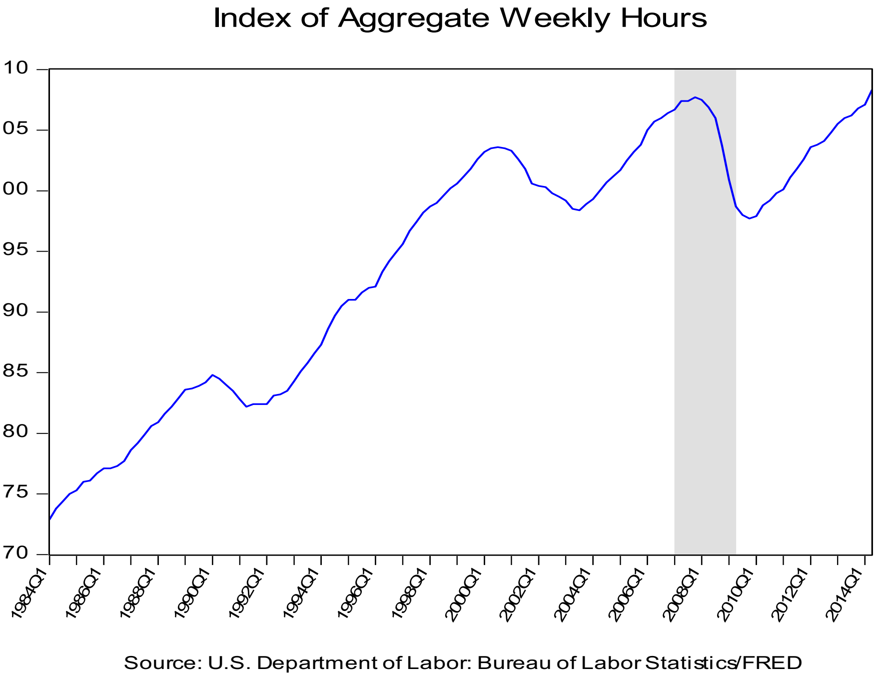


Figure 2.19: Index of Aggregate Weekly Hours (U.S.)

Regarding the price of labor, the level of wages, measured in this case by the compensation received by all employees, from Figure 2.20 we can observe that the outbreak of the financial crisis in 2007 seems to have accelerated the rate at which the wages are falling, inducing a break in the upward trend that has been verified since the first quarter of 1984. It is important to stress, however, that this break only occurred after the second quarter of 2008, when the financial crisis had already spread to the real sectors of the economy.

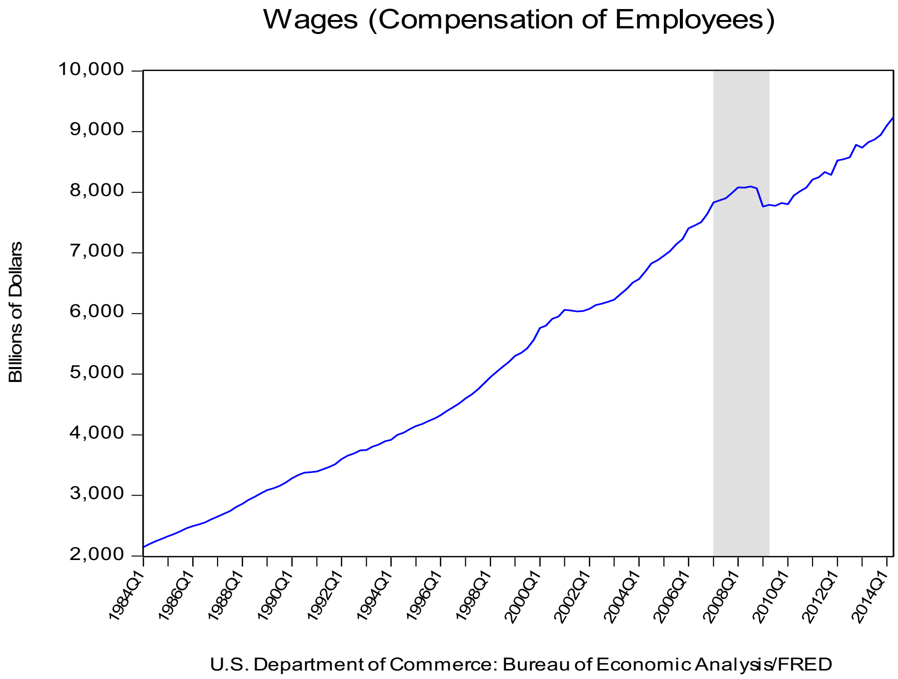


Figure 2.20: Nominal Wages (U.S.)

Using the method used by Jermann and Quadrini (2012) [8] I recalculated the capital stock series to include the period 2010.III-2014.II to their original sample. From figure 2.21 we can observe that the capital stock of the economy has been exhibiting an increasing path since the middle 1980's, but the recent 2007 financial crisis caused a trend break in the upward path, at least after the second quarter of 2008. The depressing effects of the financial shock lasted for several years after the initial impact, but the capital stock quickly recovered its upward trend until the first quarter of 2014.

[8]See section 4.3 for the complete methodology.

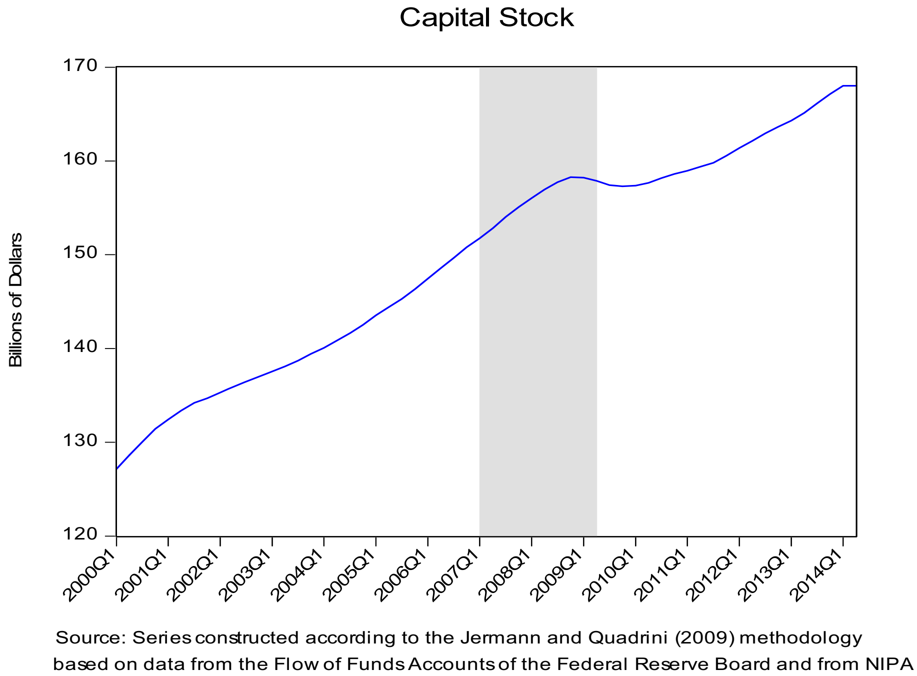


Figure 2.21: Capital Stock (U.S.)

Following the same methodology, I also reconstructed the debt stock to include the subperiod 2010.II-2014.II. From Figure 2.22, it is clear that the debt stock had a certain delayed response to the financial crisis, since it steadily increased during that period until it drop suddenly at the end of 2008. But this beahvior is related with the phenomenon already described in the previous section that claims that bank credit actually has increased during the financial crisis and only started to drop by the end of 2008 and beginnings of 2009. Although by the beginning of 2010 the debt stock had already started to recover its steady upwards trend, the financial crisis period can be considered a trend break.

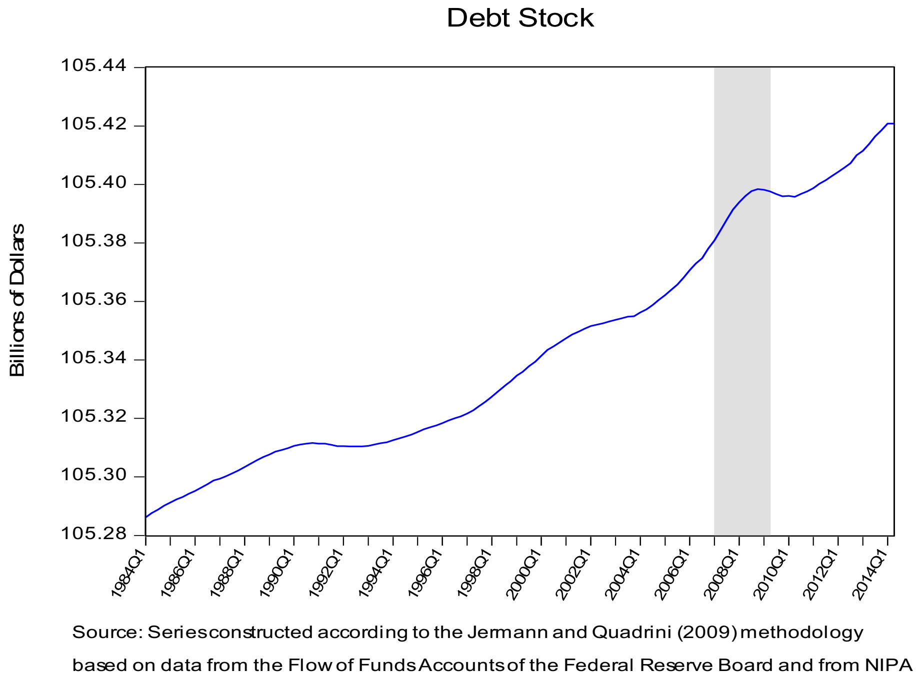


Figure 2.22: Debt Stock (U.S.)

From Figure 2.23 we can analyse what happened during the financial crisis in terms of technological advance and productivity of the U.S. economy, as measured by the total factor productivity [9]. It is possible to observe that the volatility of this variable has increased substantially over the last thirty years, especially after the hit of the dot-com bubble of the early 2000's. Throughout the official financial crisis period the total factor productivity registered a huge drop in relation to its mean, achieving a negative peak of approximately -0.02% in percent deviations from the trend in the last quarter of 2009, the lowest since the 1984. However, that descending path had already started at the end of 2004, although in that time period the variable had reached a positive peak.

[9]This variable is the productivity shock computed as a standard Solow residual using the Jermann and Quadrini (2012) approach, updating their 1984.I-2010.II sample until the second quarter of 2014.

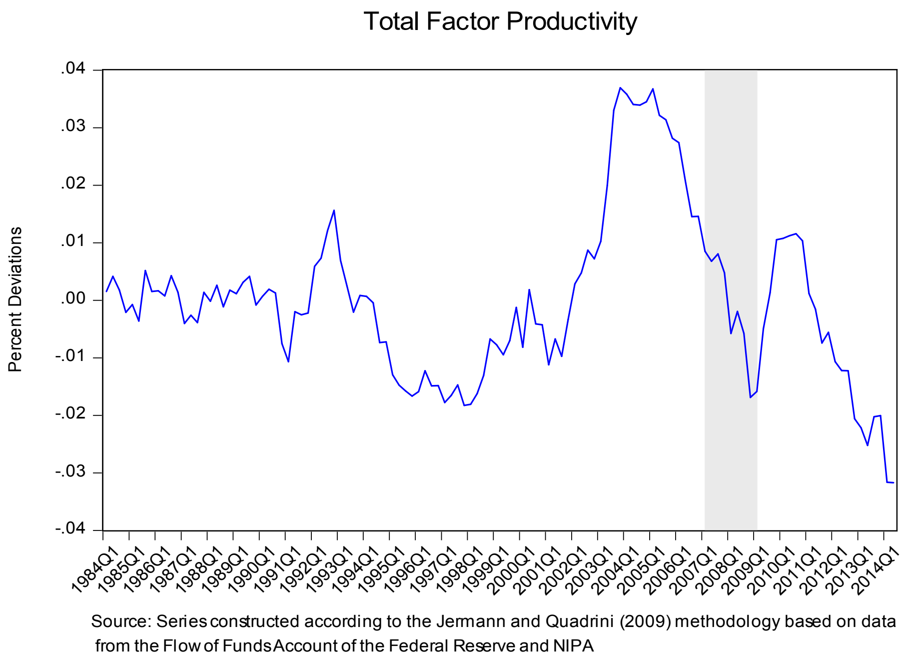


Figure 2.23: Total Factor Productivity (U.S.)

In terms of prices, the inflation rate (see Figure 2.24) registered the lowest peak (approximately −8.9%) throughout the whole time interval considered (1984.I-2014.II) precisely in the fourth quarter of 2008, in the peak of the financial crisis. This is clearly an outlier, since the inflation rate in the U.S. gravitated around its average of approximately 3% without major deviations, although its volatility has increased since the beginning of the XXI century. This sudden and huge drop in the inflation rate during the financial crisis may reflect the steep fall in aggregate demand, mainly through private consumption and investment, as can be observed in Figure 2.11.

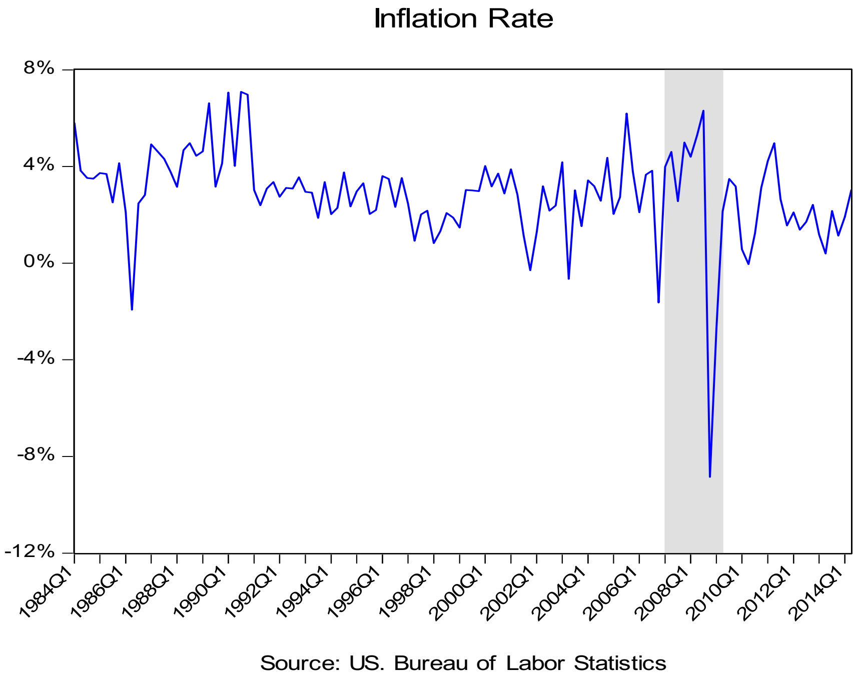


Figure 2.24: Inflation Rate (measured by CPI) in the U.S.

Figure 2.25 plots a variety of types of interest rate data with different maturities (Federal Funds rate, 1-month LIBOR, 1-month AA financial commercial paper rate, 1-month and 1-year Treasury Constant Maturity rates, and 3-months Treasury Bill). These figures show that while spreads have certainly widened, the level of interest rates of various types of borrowing are well below levels in recent non-crisis years. From the beginning of 2004 up until the first quarter of 2007, the major interest rates references of the US financial markets exhibited a steep increase, but as soon as the subprime crisis started affecting markets, the level of these interest rates plummeted heavily, although at different rhythms. Before the official financial crisis period ended, all these interest rates were below 1%, and continued to shrink further until the end of the sample (2014.II), to levels close to 0%, which resembles closely a liquidity trap situation, or at least implies that the zero lower bound constraint imposed by the central bank became active.

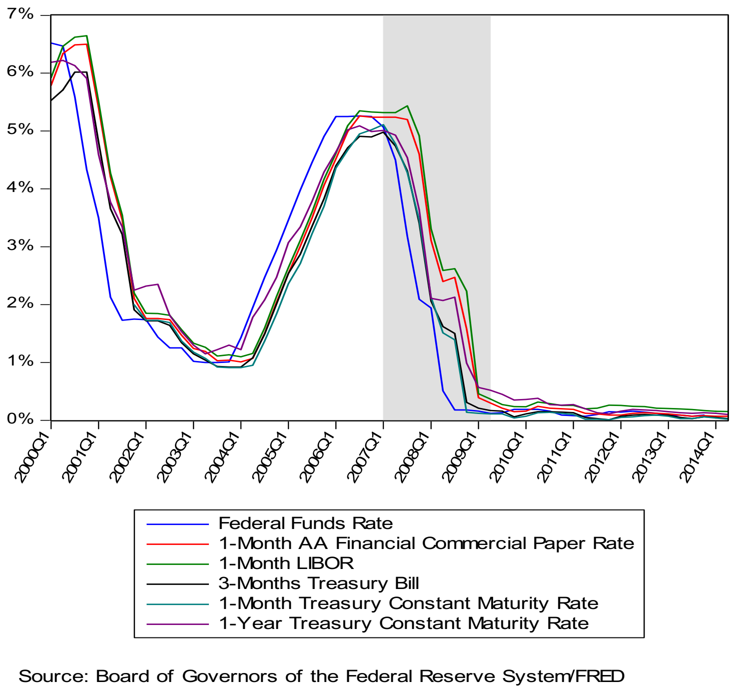


Figure 2.25: Nominal Interest Rates (U.S.)

In order to infer about how the interest rates' behaviour during the financial crisis was affected by inflation fluctuations, it is important to inspect the Treasury Inflation-Protected Security (TIPS) interest rates presented in Figure 2.26. TIPS are securities issued by the U.S. Treasury whose principal is indexed to the seasonally unadjusted Consumer Price Index (CPI) and the interest rate of semi-annual coupon payments is fixed. [10] TIPS are considered to be one of the few risk-free financial instruments offering a full hedge against high inflation to investors, by indexing the return to the price level. For these securities, the principal increases with inflation and decreases with deflation.In other words, when the price level increases investors receive higher payments, and, in case of deflation, investors are protected with a floor that guarantees them a payment of at least the original principal value. When the security matures, the U.S. Treasury pays the original or adjusted principal, whichever is greater. Therefore, by inspecting Figure 2.26, we observe that in the pre-crisis period the 5-year and 10-year TIPS interest rates level were close to

[10]The U.S. Treasury started the issuance of these debt instruments in 1997 with two main purposes: to fully protect investors from inflation risk (and consequent losses in purchasing power) and to provide daily data on future inflation market expectations to the Federal Reserve.

the $1\% - 2\%$ interval, also followed by nominal assets interest rates (2.25). In the first quarter of 2008, TIPS interest rates suffered its first major breakdown (especially the 5-year TIPS interest rate), but since TIPS are indexed to inflation, the associated interest rates recovered quickly, reaching a new peak in the last quarter of 2008. However, it was precisely during that period that the inflation rate suffered its major fall, and consequentially the TIPS interest rates plummeted heavily from that point on, reaching negative values from the end of 2010 onwards. We can better understand and interpret the relationship between TIPS interest rates and inflation by defining breakeven inflation. For a given maturity, the spread between the yields of nominal securities and TIPS can be interpreted as an inflation expectation, also known as breakeven inflation,because it represents the level of inflation that, if realized, would provide the same return to investors in nominal and inflation-indexed securities, making the investors indifferent between these different Treasury securities. Using the Fisher relationship the breakeven inflation can be interpreted as the market expectation of the average annual inflation today and the maturity of the securities.

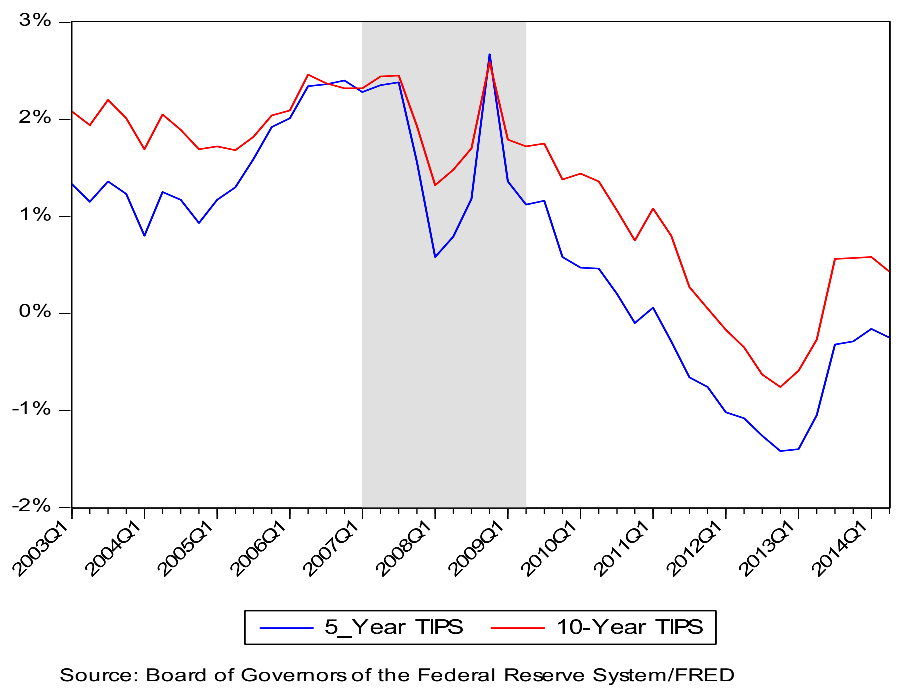


Figure 2.26: Treasury Inflation-Indexed Security (TIPS) Interest Rates

Figure 2.27 plots the joint evolution of equity payout and debt repurchases also updated from 1984.I until 2014.II. The two patterns identified by Jermann and Quadrini (2012) still hold on after the 2008 financial crisis: the two series maintain a strong negatively correlation with each other and the tendency to equity payouts increase in booms while debt repurchases increase during or around recessions also persists. However, from this figure it is also very clear the inversion in the path of the two variables that occurred at the end of the financial crisis, signalling the end of the most financial distress period.

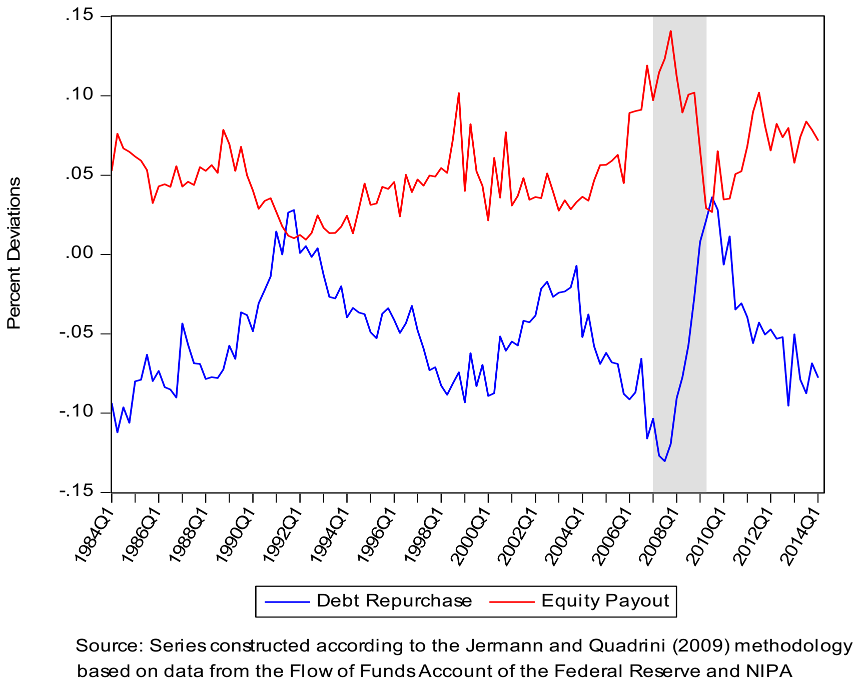


Figure 2.27: Equity Payout and Debt Repurchase

2.4 Calculations of the Productivity and the Financial Shocks Series

2.4.1 General Overview of the Model

In this paper I establish a comparison between empirical evidence based on my analysis of the macroeconomic data available for the U.S. before, during and after the collapse of financial system in the U.S. in 2008, and the results provided by the simulation of the Jermann and Quadrini (2012) model for the same time period.

Therefore, it is important to present and explain the main equations and mechanisms behind the Jermann and Quadrini (2012), especially those that are directly related with the two shocks that I want to re-estimate and simulate in the present paper, the productivity and the financial shock. In this section I provide that brief characterization of the Jermann and Quadrini (2012) framework, starting by characterizing firms (since their optimization problem is constrained by the enforcement constraint, from which the financial shock series is constructed), and then I proceed to characterize households.

2.4.1.1 Firms

There is a continuum of firms, in the $[0, 1]$ interval. The production function is characterized by a standard Cobb-Douglas specification given by $F(z_t, k_t, n_t) = z_t k_t^{\theta} n_t^{1-\theta}$, where z_t is the stochastic level of productivity, k_t is the input of capital and n_t is the input of labor. It is assumed that z_t is governed by a stochastic autoregressive process of order 1 (AR(1)): $z_t = \rho_z z_{t-1} + \varepsilon_z$, where ε_z is assumed to be i.i.d., with the mean equal to zero and with an homoescedastic variance equal to σ_z. Innovations to z_t are defined as the productivity shocks in this framework, and they affect all firms. The capital stock k_t is chosen at time $t-1$ and therefore predetermined at time t, and the input of labor n_t is chosen at time t.

The law of motion of capital is given by $k_{t+1} = (1-\delta)k_t + i_t$, where i_t is investment and δ is the depreciation rate.

It is assumed that firms use both equity d_t and debt b_t in order to finance themselves. In this model, one of the main features is the pecking order in the financial decision of firms between equity and debt. Debt is preferred to equity because of its tax advantage, but the firms' ability to borrow of limited by an enforcement constraint which is subject to random disturbances, designated as "financial shocks". Given that r_t is the net interest rate, the effective gross interest rate for the firm is $R_t = 1 + r_t(1 - \tau)$, where τ denotes the tax benefit. This tax deduction enjoyed by firms is paid for by a lump-sum tax on households. The tax advantage also implies that the aggregate amount of debt B_t will always be strictly positive.

In order to finance working capital, in addition to the debt, b_t, firms can raise funds with an intra-period loan l_t. According to Jermann and Quadrini (2012), "working capital is required to cover the cash flow mismatch between the payments made at the beginning of the period and the realization of revenues" (after production). Therefore, according to Lucas timing, the intra-period loan is repaid at the end of the period and there is no interest, by assumption. The assumption of the existence of this intra-period loan is a short cut to the fact that firms carry "cash" or "liquidity" to the next period. The loan is then used to pay the equity holders (including dividends) and to finance working capital (wages and investment), implying that the payments of dividends comes from previous periods earnings. Firms also issue equity shares s_t to its shareholders at the market price p_t.

Firms start the period with intertemporal debt b_t. Before producing they choose labor, n_t, investment, $i_t = k_{t+1} - (1 - \delta)k_t$ and equity payout, d_t, and they issue new intertemporal debt, b_{t+1}. Firms also contract an intra-period loan l_t, since the payments to workers, suppliers of investments, shareholders and bondholders are made before the realization of revenues. That contract defines the equivalent of a cash-in-advance (CIA) condition:

$$l_t = w_t n_t + i_t + d_t + b_t - b_{t+1}/R_t \tag{2.1}$$

The firm's budget constraint is the following:

$$b_t + w_t n_t + k_{t+1} + d_t = (1-\delta)k_t + F(z_t, k_t, n_t) + b_{t+1}/R_t \tag{2.2}$$

Using the previous two conditions follows that the intra-period loan is equal to the firm's production function/revenues, i.e. $l_t = F(z_t, k_t, n_t)$. However, this is only true as long as there is the tax benefit ($\tau > 0$).

Since firms can default on their obligations and divert some of its own resources (specifically, the amount of liquidity available at period t, which corresponds to the intra-period loan $l_t = F(z_t, k_t, n_t)$), the ability to borrow in both time horizons is constrained by the limited enforceability of debt contracts. According to Jermann and Quadrini (2012), "the decision to default arises after the realization of revenues but before repaying the intra-period loan". At this stage the firm holds a total amount of liabilities given by the intra-period loan plus the new intertemporal debt, i.e. $l_t + b_{t+1}/R_t$. In case of default, the only asset available for liquidation is the physical capital k_{t+1}, since firms can easily divert the total amount of liquidity available at that period.

The renegotiation process between the firm and the lender [11] in the event of default, in this economy, takes place without the introduction of a well defined system of financial intermediation. If the firm defaults, the lender acquires the right to liquidate the firm's capital. It is assumed that at the moment of contracting the loan the liquidation value of physical capital k_{t+1} is uncertain. With probability ξ_t the lender will be able to recover the whole value k_{t+1} but with probability $1 - \xi_t$ the recovery value is zero. Neither the lender nor the firm are able to observe the liquidation value before the actual default. Therefore, these two cases are considered separately to determine the renegotiation outcome. In order to do so, it is assumed that the firm has all the bargaining power in the renegotiation and the lender gets only the threat value.

Therefore, the enforcement constraint faced by firms is given by:

[11] For a complete description of the renegotiation process between firms and lenders, see Jermann and Quadrini (2012).

$$\xi_t\left(k_{t+1} - \frac{b_{t+1}}{1+r_t}\right) \geq l_t \tag{2.3}$$

When this constraint is binding, higher debt, either intertemporal or intratemporal, tightens the enforcement constraint, and a higher stock of capital has the opposite effect. When this enforcement constraint does not bind, that is, when $l_t < k_{t+1} - b_{t+1}/(1+r_t)$, it imposes a liquidity cost that acts in the economy as an inflation tax: if the disposable amount of liquidity $l_t = F(z_t, k_t, n_t)$ given by the production is not sufficient to make the payment that leaves the lender indifferent between liquidation and keeping the firm in operation, then that implies that the firm will not be able to finance working capital imposing an implicit tax on its workers, investors, shareholders and bondholders.

The probability ξ_t is stochastic and follows the process $\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_\xi$. It is common for all firms and depends on market conditions. We can interpret ξ_t as the probability of non-default by the firms. Stochastic innovations to this variable will be considered the "financial shocks" of this model, since it affects the tightness of the enforcement constraint, therefore affecting the borrowing capacity of the firm. Hence, so far, we have two sources of aggregate uncertainty in this model: productivity shocks, z_t, and financial shocks, represented by ξ_t. Since there are no idiosyncratic shocks, I will focus on the symmetric equilibrium solved for a representative firm.

From this enforcement constraint, it is possible to conclude that whether the financial shock affects employment depends crucially on the trade-off between debt and equity when the firm has to decide the composition of its financial structure. In other words, the Modigliani-Miller theorem asserting the irrelevance of firms asset structures does not apply in this economy when the enforcement constraint binds. [12]

To formalize the rigidities affecting the substitution between debt and equity, and in order to capture the frictions associated with paying dividends, I assume

[12] Jermann and Quadrini (2012) provide a more detailed explanation on how to achieve this conclusion using the enforcement constraint as a starting point.

that the firm's payout is subject to a quadratic adjustment cost, following Jermann and Quadrini (2012) framework. Given that d_t is equity payout, the actual cost for the firm is given by:

$$\varphi(d_t) = d_t + \kappa(d_t - \overline{d})^2, \tag{2.4}$$

Where $\kappa \geq 0$, and $\overline{d}$ is a parameter equal to the long-run payout at the steady state.

According to Jermann and Quadrini (2012), the equity payout can be interpreted as a way of modelling the flexibility and "the speed with which firms can change the source of funds when financial conditions change, i.e. it can be interpreted as the financial cost in which firms have to incur due to the trade-off between equity and debt". In order to change the composition of its portfolio, the firm must incur in legal and accounting costs associated with issuing and repurchasing equity shares, as well as costs associated with adjusting their equity payouts to shareholders. The parameter κ is a measure of the scale of the adjustment cost of the firm's payouts to shareholders. The quadratic cost function implies that the adjustment cost is increasing in the deviation of today's equity payout from its long term steady-state payout target. The convex functional form of the adjustment cost given here is meant to mimic the empirical evidence that the preferences of managers turn to dividend smoothing over time.

The parameter κ is determinant to evaluate the impact of financial shocks over the rest of the economy. When $\kappa = 0$, the economy is almost equivalent to a frictionless economy, and in this case, "debt adjustments triggered by the enforcement constraint can be costlessly accommodated through changes in firm equity", according to Jermann and Quadrini (2012). When $\kappa > 0$, the substitution between debt and equity becomes costly and the adjustment to a different financial structure becomes slower, affecting the firm's production decisions. This is the main reason why financial shocks will produce non-negligible short-term effects on the production decision of firms.

Besides issuing non-contingent bonds b_t, firms also issue equity shares s_t, at the market price p_t. Let us denote the total amount of equity payout received from owning shares as d'_t (and the net equity payout simply as d_t), in order to solve the firm's optimization problem. Then, d'_t can be defined as $d'_t = s_t d_t + p_t(s_t - s_{t+1})$, as the sum of the total amount of dividends distributed by the shareholders in period t and the total net amount of share repurchases available in the same period.

The individual state variables are the capital stock, k, the debt, b, and the equity shares, s. The aggregate states are the equity payout d and the input of labor, n.

Given this, the firm's optimization problem is the following:

$$\max_{\{d_t, n_t, k_{t+1}, b_{t+1}, s_{t+1}\}_{j=0}^{\infty}} \left\{ E_t \left[\sum_{j=0}^{\infty} m_{t+j} d_{t+j} \right] \right\} \tag{2.5}$$

subject to:

$$(1-\delta)k_t + F(z_t, k_t, n_t) - w_t n_t + \frac{b_{t+1}}{R_t} + p_t s_t = b_t + \varphi(d'_t) + k_{t+1} + p_t s_{t+1}$$

$$\xi_t \left(k_{t+1} - \frac{b_{t+1}}{1+r_t} \right) \geq F(z_t, k_t, n_t)$$

Where m_{t+j} is the stochastic discount factor, which will be defined later in the hoseholds' optimization problem. This variable, along with the wage and the interest rate are all determined in the general equilibrium and are all taken as given by the representative firm.

The first-order conditions of this optimization problem for d_t, n_t, k_{t+1} and b_{t+1} and s_{t+1} are given by the following conditions, after substituting $\varphi_d(d'_t) = s_t + 2\kappa s_t(d'_t - \bar{d}) = s_t + 2\kappa s_t(s_t d_t + p_t(s_t - s_{t+1}) - \bar{d})$ back into each one of these conditions:

$$F_n(z_t, k_t, n_t) = w_t \left(\frac{1}{1 - \mu_t(1 + 2\kappa(d'_t - \bar{d}))} \right) \tag{2.6}$$

$$E_t m_{t+1} \left(\frac{1 + 2\kappa(d'_t - \bar{d})}{1 + 2\kappa(d'_{t+1} - \bar{d})} \right) \left[(1-\delta) + \Big(1 - \mu_{t+1}(1 + 2\kappa(d'_{t+1} - \bar{d}))\Big) F_k(z_{t+1}, k_{t+1}, n_{t+1}) \right] + \xi_t \mu_t (1 + 2\kappa(d'_t - \bar{d})) = 1 \tag{2.7}$$

$$R_t E_t m_{t+1}\left(\frac{1+2\kappa(d'_t-\bar{d})}{1+2\kappa(d'_{t+1}-\bar{d})}\right)+\xi_t\mu_t(1+2\kappa(d'_t-\bar{d}))\left(\frac{R_t}{1+r_t}\right)=1 \tag{2.8}$$

$$E_t m_{t+1}\left(\frac{1+2\kappa(d'_t-\bar{d})}{1+2\kappa(d'_{t+1}-\bar{d})}\right)p_{t+1}=p_t \tag{2.9}$$

According to Jermann and Quadrini (2012), each one of these conditions have an intuitive interpretation that provides further insights into the model. The first condition (2.6) determines optimality for labor, where the left hand side of the equation is the marginal productivity of labor and the right hand sight is the marginal cost of labor, as usual. However, in this case the marginal cost differs from the standard expression because the wage rate is augmented by a wedge that depends on the 'effective' tightness of the enforcement constraint, given by $\mu\varphi_d(d)$. Due to this wedge, a tighter constraint increases the effective cost of labor and reduces its demand, decreasing employment. Therefore, this friction in labor demand constitutes the main channel of transmission of financial shocks to the real sector of the economy.

From the second condition (2.7) it is not immediately clear whether the determination of the optimal level for capital in this economy depends or not directly in the interest rate r_t. In this case, the marginal productivity of capital is also negatively influenced by the wedge $\mu_t\varphi_d(d_t)$, which implies that the tighter the enforcement constraint and the higher the cost of equity payout, the lower will be the stock of capital demanded in equilibrium. Solving the third condition (2.8) in order of $\xi_t\mu_t\varphi_d(d_t)$ and substituting the result into the second condition, we get an expression which resembles more closely the standard condition for the firm's demand for capital:

$$Em_{t+1}\left(\frac{\varphi_d(d_t)}{\varphi_d(d_{t+1})}\right)\Big[(1-\mu_{t+1}\varphi_d(d_{t+1}))F_k(z_{t+1},k_{t+1},n_{t+1})-(r_t+\delta)\Big]=\frac{R_t-(1+r_t)}{R_t} \tag{2.10}$$

From this condition shows that the determination of capital in equilibrium does not depend directly on ξ_t, and therefore its innovations (the financial shocks) do not have a direct impact over the stock of capital. However, the financial shocks have an indirect effect over the demand for capital through two channels: the multiplier μ_t and the interest rate R_t. The higher the μ_t, i.e., the tighter is the enforcement constraint, the higher the wedge over the marginal productivity of capital, and consequentially the smaller the demand for capital. If we consider the special case in which the cost of payout is zero, i.e., $\kappa = 0$, this implies that $\varphi_d(d_t) = \varphi_d(d_{t+1}) = 1$. Applying this condition, (9) and (19) become, respectively:

$$Em_{t+1}\left[(1-\mu_{t+1})F_k(z_{t+1},k_{t+1},n_{t+1})\right] + \xi_t\mu_t = 1$$

$$Em_{t+1}\left[(1-\mu_{t+1})F_k(z_{t+1},k_{t+1},n_{t+1}) - (r_t+\delta)\right] = \frac{R_t-(1+r_t)}{R_t}$$

As we can see from the second expression, even in the absence of the cost of equity payout, the demand for capital is always pinned down by the tightness of the enforcement constraint.

2.4.1.2 Households

The households sector is composed by a continuum of homogeneous households. According to Jermann and Quadrini (2012) assumptions, "households are the owners (shareholders) of firms. In addition to equity shares, they also hold non-contingent bonds issued by firms".

Households maximize their lifetime expected utility subject to their budget constraint. The complete representation of the problem is therefore given by

$$\max_{\{n_t,b_{t+1},s_{t+1}\}_{j=0}^{\infty}} \left\{E_t\left[\sum_{j=0}^{\infty}\beta^j U(c_{t+j},n_{t+j})\right]\right\} \tag{2.11}$$

subject to:

$$w_t n_t + b_t + s_t(d_t+p_t) = \frac{b_{t+1}}{1+r_t} + s_{t+1}p_t + c_t + T_t$$

where where E_0 is the expectation operator conditional on the information set at time $t = 0$, Ω_0, c_t is consumption, β is the discount factor, and $T_t = B_{t+1}/[1 + r_t(1-\tau)] - B_{t+1}/(1+r_t)$ are lump-sum taxes financing the tax benefit of debt and firms.

After defining the Lagrangian of the problem, the first order conditions with respect to n_t, b_{t+1} and s_{t+1} are:

$$w_t U_c(c_t, n_t) + U_n(c_t, n_t) = 0 \tag{2.12}$$

$$U_c(c_t, n_t) - \beta(1 + r_t) E U_c(c_{t+1}, n_{t+1}) = 0 \tag{2.13}$$

$$U_c(c_t, n_t) p_t - \beta E(d_{t+1} + p_{t+1}) U_c(c_{t+1}, n_{t+1}) = 0 \tag{2.14}$$

$$TVC : \lim_{t \to \infty} E_t \beta^t \lambda_t b_t = 0 \tag{2.15}$$

The first condition determines the supply of labor and the second condition is the Euler equation that determines the interest rate. The third condition determines the price of shares, which states that, adjusted for dividends and discounting, the share price follows a first-order univariate Markov process and that no other variables can Granger cause the share price. Using forward substitution in this last condition I get the following condition:

$$p_t = E_t \sum_{j=1}^{\infty} \left(\frac{\beta^j . U_c(c_{t+j}, n_{t+j})}{U_c(c_t, n_t)} \right) d_{t+j}$$

According to Jermann and Quadrini (2012), this last equation confirms that firms' optimization is consistent with households' optimization. Therefore, "the stochastic discount factor is $m_{t+j} = \beta^j U_c(c_{t+j}, n_{t+j})/U_c(c_t, n_t)$".

2.4.2 Parameterization

As in Jermann and Quadrini (2012), the parameters are grouped into two sets: one that includes parameters that can be calibrated using steady state targets, and the other one which includes parameters that cannot be calibrated using those targets.

In the case of the first set of parameters, the majority of the calibrated values follows closely the standard RBC related literature for the US historical data. Table 2.1 presents all the calibrated values for this set of parameters.

The time discount parameter is defined as $\beta = 0.9825$, implying that the annual steady state return from holding shares is 7.32 percent, according to Jermann and Quadrini (2012) estimations. The utility function has the functional form $U(c, n) = ln(c) + \alpha.ln(1 - n)$, where $\alpha = 1.8834$ is chosen to have steady state hours equal to 0.3. The Cobb-Douglas parameter in the production function is set to $\theta = 0.36$ and the depreciation to $\delta = 0.025$. The mean value of z is normalized to 1. These values are standard and the quantitative properties of the model are not very sensitive to this first group of parameters.

The tax wedge is set to $\tau = 0.35$, which corresponds to the benefit of debt over equity if the marginal tax rate is 35 percent. This parameter is very important for the model because it determines whether the enforcement constraint is binding or not. In fact, this value of τ and the all the remaining parameterizations of the model are set in order to make the enforcement constraint always binding in the simulations (except when we set $\tau = 0$ in order to simulate the model without the interest rate benefit).

Finally, the mean value of the financial variable, or the enforcement parameter, $\bar{\xi}$, is chosen to have a steady state ratio of debt over quarterly GDP equal the historical value, over the whole span of the sample considered. In the original sample, this corresponded to the average ratio over the first quarter of 1984 until the second quarter of 2010 for the nonfinancial business sector based on data from the Flow of Funds (for debt) and National Income and Product Accounts (for business GDP). The required value was set to $\bar{\xi} = 0.1634$, in the original Jermann and Quadrini (2012) sample. However, for the main objectives of this paper, the calibration of this particular parameter is crucial to the main findings, since the calibrated value of $\bar{\xi}$ determines the construction of the financial shock, and therefore its conditioned on the particular time span chosen to compute the value of this mean. For that reason, and since Jermann and Quadrini (2012) haven't specified the exact method used to calibrate this parameter, I will describe in detail in section 4.3 how I calibrated this parameter, and under which assumptions.

Table 2.1: Parameters set with Steady State targets

Description	**Parameters**
Discount factor	$\beta = 0.9825$
Tax advantage	$\tau = 0.3500$
Utility parameter	$\alpha = 1.8834$
Production technology	$\theta = 0.3600$
Depreciation rate	$\delta = 0.0250$
Payout cost parameter	$\kappa = 0.1460$

Source: Jermann and Quadrini (2012)

According to Jermann and Quadrini (2012), "the parameters that cannot be set with steady state targets are those determining the stochastic properties of the shocks and the cost of equity payout - the parameter κ. In a steady state the stochastic properties of the shocks do not matter and the equity payout is always equal to the long-term target, therefore an alternative procedure was followed to construct the series of productivity and financial shocks". For that reason, a detailed explanation of the calibration of the remaining parameters (set with or without steady state targets) and the method to estimate the time series for the two stochastic shocks is provided in the next section.

2.4.3 Data and Methodology

The data collected in this paper includes the same database used by Jermann and Quadrini (2012), updated until the second quarter of 2014 (the original sample ranged from the first quarter of 1952 until the second quarter of 2010, although the authors focused on the interval 1984.I-2010.II to perform the simulations of their model. Therefore, my sample includes U.S. quarterly data which covers the period 1952.I - 2014.II.

I focus essentially on U.S data since the Great Recession originally started in that country with the subprime financial crisis, and in order to identify and isolate the effects of the financial shock over the majority of the variables studied. Inside the full sample, I focus essentially on the period between 1984.I - 2014.II, because it includes the time range originally analyzed by Jermann and Quadrini (2012) (1984.I - 2010.II), and upon which they calibrated their model in order to perform simulations. One of the major exercises of this paper is precisely to compare the main changes that including this observations to the original sample caused in the calibration of the model, and also compare the performance of the model in anticipating and replicate the empirical behavior, in terms of impulse responses and simulations of the major aggregates.

I also emphasize the period 2010.III - 2014.II, because it is precisely the time period which updates the original Jermann and Quadrini (2012) sample, and corresponds to the post-2008 financial crisis period but coincides with the peak of the so called Great Recession (the period in which the initial financial crisis had already transformed into a worldwide phenomenon that affected all the sectors of the economy.

All time series included in this study, financial and real, are seasonally adjusted. In order to obtain the data, the major sources are the Flow of Funds Accounts of the Federal Reserve Board, the Federal Reserve Economic Data (FRED) database of the Federal Reserve Bank of Saint Louis, the Bureau of Labor Statistics, and the National Income and Product Accounts (NIPA) database.

In terms of methodology, the initial step is to construct series for productivity and financial shocks, following the approach of Jermann and Quadrini (2012), but extending their original sample (1952.I-2010.I) until 2014.II, and compute series for capital stock, debt stock, equity payout, debt repurchases and total factor productivity, along with the most important statistical moments of the each series. Then I simulate the Jermann and Quadrini (2012) model and compute statistical moments and IRF's for the main macroeconomic variables. Finally, I compare the results.

From the general overview of the model presented in Section 4.1, the productivity shock is constructed as a total factor productivity (TFP) series derived after log-linearizing the production function $y_t = z_t k_t^{\theta} n_t^{1-\theta}$:

$$\hat{z}_t = \hat{y}_t - \theta \hat{k}_t - (1-\theta)\hat{n}_t \tag{2.16}$$

The financial shock series is constructed from the enforcement constraint $\xi_t \left(k_{t+1} - b_{t+1}^e\right) = y_t$, where $b_{t+1}^e = b_{t+1}/(1+r_t)$ is the end of period liability. The linearized version of this constraint is:

$$\hat{\xi}_t = \phi_k \hat{k}_{t+1} + \phi_b \hat{b}_{t+1}^e + \hat{y}_t, \tag{2.17}$$

where $\phi_k = -\bar{\xi}\bar{k}/\bar{y}$ and $\phi_b = \bar{\xi}\bar{b^e}/\bar{y}$ are constructed using steady state targets. The hat sign denotes percent deviations from the steady state and the bar sign denotes steady state values.

Recalculating the times series for these two shocks ($\hat{z}_t$ and ϕ_k) according to the extended sample 1984.I-2014.II, implies that we need to reestimate parameters $\bar{\xi}$, ϕ_k, ϕ_b and also the debt/output and capital/output ratios. However, when adopting this method to conduct that reestimation process, a serious problem emerged when collecting the data from the exact same sources used by Jermann and Quadrini (2012): I detected that their original estimation suffered from a methodological issue related to the fact the constructed TFP and financial shock series combines a nominal GDP of the total economy constructed with either total real GDP or with inputs pertaining to the private business sector only (Business Value Added) [13].

This problem was detected due to statistical discrepancies between the updated parameters and the original Jermann and Quadrini (2012) parameters needed to construct both the productivity and the financial shocks, as can be observed in

[13] This problem was later also detected and corrected by Pfeifer (2016).

Table 2.2. In other words, the estimates that I obtained for the coefficients $\bar{\xi}$, ϕ_k, ϕ_b are not exactly equal to the Jermann and Quadrini (2012) estimates computed for the same time interval (1984.I-2010.II).

Table 2.2: Productivity and Financial Shock Parameters Estimations - Sample 1984.I-2010.II

	Original J&Q Data	**Updated Data**	
	Sample 1984.I-2010.II	**Method 1**	**Method 2**
$\bar{\xi}$	0.1634	0.2111	0.4707
Debt/output	3.36	5.8859	2.5223
Capital/output	9.4799	10.6232	4.6468
ϕ_k	-1.5489	-0.5601	-1.2489
ϕ_b	0.5489	0.3231	0.7205

Source: Author's own calculations.

Therefore, I adopted two different methods to estimate these parameters based on two different approaches Jermann and Quadrini (2012) when computing the Nominal GDP used in the calculations of the two shocks measures:

Method 1:

$Nominal\ GDP = Real\ GDP \times Business\ Price\ Index$

Method 2:

$Nominal\ GDP = Business\ Value\ Added \times Business\ Price\ Index$

Where *Real GDP* is the Real Gross Domestic Product in billions of chained (2009) dollars and seasonally adjusted at annual rates, extracted from the Bureau of

Economic Analysis (NIPA); *Business Value Added* is the total value added accumulated in the business sector, also extracted from NIPA; and *Business Price Index*, which is the price index for the business sector also added from NIPA.

Table 2.3 sums up the estimates obtained for the updated sample between Sample 1984.I-2010.II according with these two approaches:

Table 2.3: Productivity and Financial Shock Parameters Estimations - Sample 1984.I-2014.II

	Sample 1984.I-2014.II	
	Updated Data	
	Method 1	**Method 2**
$\bar{\xi}$	0.2229	0.5093
Debt/output	6.1334	2.5159
Capital/output	10.6186	4.4795
ϕ_k	-0.0054	-1.344
ϕ_b	0.2903	0.8277

Source: Author's own calculations.

We can then use the above equation to construct the $\hat{\xi}_t$ series once we have empirical measurements for the end of period capital, $\hat{k}_{t+1}$, the end of period liabilities, $\hat{b}^e_{t+1}$, and output $\hat{y}_t$. Following is the description of how these series are constructed.

The series for the Capital Stock k_t were constructed as follows:

$$k_{t+1} = k_t - Depreciation + Investiment \tag{2.18}$$

where *Depreciation* is measured as "Consumption of fixed capital in nonfinancial corporate business" plus "Consumption of fixed capital in nonfinancial noncorporate business", *Investment* is measured as "Capital expenditures in nonfinancial business", and all series were extracted from the Flow of Funds Accounts of the

Federal Reserve Board. Both variables are deflated by the price index for business value added from NIPA. The initial k_t is chosen so that the capital-output ratio in the business sector does not display any trend during the sample period 1052-2014 (the full complete sample in years). This procedure undertaken by Jermann and Quadrini (2012) is not relevant for these results based on the subperiod 1984-2014, since the initial value of k_t is important only for the early dates. The $\hat{k}_{t+1}$ series used in equation (2.4.3) is constructed by linearly detrending the logarithm of k_{t+1} over the period 1984.I-2014.II.

The Debt Stock is constructed using the equation:

$$b_{t+1}^e = b_t^e + Net\ New\ Borrowing \tag{2.19}$$

In this case, the variable $b_{t+1}^e = b_{t+1}/(1+r_t)$ is used instead of simply b_{t+1} because this is the model equivalent of the end-of-period debt reported in the data. *Net New Borrowing* is measured by "Net increase in credit markets instruments of nonfinancial business", also from NIPA. Since the constructed stock of debt is measured in nominal terms, we deflate it by the price index for business value added from NIPA. Contrary to Jermann and Quadrini (2012) [14], the initial (nominal) stock of debt was set to 105.232, which was the value reported at the extraction date in the same balance sheet data from the Flow of Funds used by authors. However, since this recursion was started in 1952, the initial value is not important for the results of the paper based on the subperiod 1984-2014. The series for the stocks of debt were not directly used since they are not seasonally adjusted. The $\hat{b}_{t+1}^e$ series used in equation was, therefore, computed by linearly detrending the log of b_{t+1}^e over the sample period 1984.I-2014.II.

The construction of *Output* series $\hat{y}_t$ is the point of my analysis where I divide the calculations into two different approaches: first, following Jermann and Quadrini

[14] Jermann and Quadrini (2012) set this value to 94.12, which was the value reported at the time their analysis was executed in the balance sheet data from the Flow of Funds Accounts of the Federal Reserve Board in 1952.I for the nonfarm financial business. This discrepancies are due to the systematic updates that the Flow of Funds perform regularly over these data sets.

(2012), I use the business value added series from NIPA; and second, I use the real GDP in billions of chained (2009) dollars from the Flow of Funds. The resulting series $\hat{y}_t$ used in equation is constructed by linearly detrending the log of y_t over the period 1984.I-2014.II.

Then, following Jermann and Quadrini (2012), it is assumed that both shocks follow an Autoregressive process of order 1 (AR(1)) process, which is estimated as a Vector Autoregression of order 1 (VAR(1)):

$$\begin{pmatrix} \hat{z}_{t+1} \\ \hat{\xi}_{t+1} \end{pmatrix} = \mathbf{A} \begin{pmatrix} \hat{z}_t \\ \hat{\xi}_t \end{pmatrix} + \begin{pmatrix} \epsilon_{z,t+1} \\ \epsilon_{\xi,t+1} \end{pmatrix} \tag{2.20}$$

where $\epsilon_{z,t+1}$ and $\epsilon_{\xi,t+1}$ are iid with standard deviations σ_z and σ_ξ, respectively.

Several VAR(1) estimations were performed for the two different approaches and for 4 different subsamples: the benchmark sample 1984.I-2014.II, the complete sample 1952.I-2014.II, the original sample used by Jermann and Quadrini (2012) 1984.I-2010.II and the subsample 2010.III-2014.II, which corresponds to the period not analyzed in the Jermann and Quadrini (2012) paper.

However, when conducting these regressions, a problem emerged: several VAR(1) estimations reported that the system are unstable, due to the presence of a unit root ($\rho_\xi > 1$), for all the time periods considered and assuming both estimating methods. The autoregressive system represented above illustrates an example of one of those unstable VAR(1) estimations, where the financial shock series $\hat{\xi}_{t+1}$ has a unit root. This poses a serious issue since it is not possible to run simulations on Dynare with an unstable system.

Example: Sample 1984.I-2010.II, Method 1:

$$\mathbf{A}= \begin{bmatrix} 0.948001 & -0.018252 \\ -0.001153 & 1.026027 \end{bmatrix}$$

In order to overcome this problem, I followed two different strategies: I forced the estimation of the VAR(1) system manually imposing $\rho_\xi = 0.99$, and I estimated the two shocks as VAR(2) processes:

$$
\begin{pmatrix} \hat{z}_{t+1} \\ \hat{\xi}_{t+1} \\ \hat{z}_t \\ \hat{\xi}_t \end{pmatrix} = \mathbf{A} \begin{pmatrix} \hat{z}_t \\ \hat{\xi}_t \\ \hat{z}_{t-1} \\ \hat{\xi}_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_{z,t+1} \\ \epsilon_{\xi,t+1} \\ \epsilon_{z,t} \\ \epsilon_{\xi,t} \end{pmatrix} \tag{2.21}
$$

where $\epsilon_{z,t+1}$, $\epsilon_{\xi,t+1}$, $\epsilon_{z,t}$ and $\epsilon_{\xi,t}$ are iid with standard deviations σ_z and σ_ξ, respectively.

The VAR(2) specification for shocks proved to be stable for almost all cases considered, both in type of method and sample size.

After obtaining these estimates, I run simulations on Dynare for all the endogenous variables included in the Jermann and Quadrini (2012) model using the updated calibration obtained with the new series for the shocks and the estimates obtained from the VAR(1) and VAR(2) systems for the two shocks. Those findings are reported in the next section.

2.5 Simulations and Findings

2.5.1 Volatility and Correlation

In order to evaluate the performance of the Jermann and Quadrini (2012) model in replicating the volatility of the main macroeconomic aggregates found in the data, Table 2.4 establishes a comparison between the standard deviations of the main variables of the model computed with empirical evidence against the standard deviations of the same variables obtained by conducting stochastic simulations for 3 different scenarios: the model where the coefficients that define the VAR(1) system of the two shocks are set with the original Jermann and Quadrini (2012) calibration; the model where the coefficients that define the VAR(1) system that characterize the two shocks are computed using Method 1; and the where the coefficients that define the VAR(2) system that characterize the two shocks are computed using Method 2. All these simulations are conducted for the period 1984.I-2014.II.

From Table 2.4, in relation to output volatility, it is clear that the stochastic simulation that most closely approximates the data is the model where the shocks stochastic processes are estimated through a VAR(1) system by using Method 1 to calibrate the parameters that define those shocks. The simulation conducted under the original Jermann and Quadrini (2012) calibration actually overestimates those fluctuations, while the simulation where the shocks processes are estimated through a VAR(2) underestimated that volatility.

In general, for the real variables (output, consumption, hours worked, wages and capital stock), the three methods underestimate the empirical findings for the standard deviation, although the model estimated using the original calibration proposed by Jermann and Quadrini (2012) is the one that presents the highest standard deviations of the three approaches. In turn, the model that uses the shocks series estimated assuming a VAR(2) stochastic system to define them is the simulation where that underestimation is the strongest. This conclusions are specially true for real wages, since the empirical standard deviation for this variable is approximately 0.187223, while for the simulation with the Jermann and Quadrini (2012) calibration is close to 0.041666, for the second simulation (using a VAR(1) system and Method 1) the standard deviation is 0.037289 and for the third simulation (using a VAR(2) system and Method 2) that value is approximately just 0.015417.

In relation to the financial variables and the shocks, however, that behavior is completely different. Although all simulations still underestimate the volatility of debt repurchases, they all overestimate by a large amount the empirical standard deviation of equity payouts (0.026741 in the data, 0.857787 in the first simulation with the Jermann and Quadrini (2012) calibration, 0.195599 in the simulation that uses the VAR(2) system and Method 2 and the highest value, 1.149209, obtained in the simulation with VAR(1) and using Method 1). Regarding the productivity shock, z_t, the third simulation that assumes a VAR(2) system for the shocks and uses Method 2 presents the closest approximation (0.016621) to the empirical standard deviation (0.014452), since the other two simulations both overestimate the standard deviation obtained from data. The same conclusion holds for the financial shock, since the standard deviation of the third simulation (0.028459) is the closest to the value of 0.038141 computed with empirical evidence.

Table 2.4: **Standard Deviation - Sample: 1984:I - 2014.II**

Variables	**Data**	**Original Simulation** J&Q VAR(1) Calibration	**Stochastic Simulation** With VAR(1) Method 1	**Stochastic Simulation** With VAR(2) Method 2
y	0.055765	0.068739	0.056966	0.02164
c	0.050827	0.038352	0.032734	0.014088
n	0.045732	0.015528	0.020834	0.009132
w	0.187223	0.041666	0.037289	0.015417
k	0.082407	0.053501	0.041944	0.015534
b	0.208294	0.073372	0.02882	0.032355
d	0.026741	0.857787	1.149209	0.195599
z	0.014452	0.052861	0.038078	0.016621
ξ	0.038141	0.112304	0.125215	0.028459

Table 2.5 reports the correlations between real GDP and the simulated series of all variables obtained from the three stochastic simulations of the model, for the period 1984.I-2014.I. As we can observe, all simulations tend to underestimate the empirical correlation between consumption and output (92.92%): the first simulation gives a correlation of 82.10% between these two variables, while the second simulation reports a correlation of approximately 88.12% and the third simulation (using a VAR(2) and Method 2 to compute nominal GDP) reports a correlation of 88.25%. However, for the majority of the variables, the simulations seem to overestimate the empirical correlation between GDP and the remaining variables. This is specially true for the first simulation that uses the original calibration from Jermann and Quadrini (2012), since that, besides consumption, the estimated correlation be-

tween GDP and all the other variables overestimates the data in all cases.

The second simulation that uses a VAR(1) system to characterize the two shocks and uses Method 1 to compute nominal GDP to construct those series also tends to overestimate the correlation between GDP and the other variables, except for the capital stock, whose correlation with GDP (11.27%) is underestimated in this case, and is the one that is closer to the empirical value value (21.25%).

The third simulation that assumes that the two shocks can be defined by a VAR(2) system and uses Method 2 presents a more mixed pattern regarding the correlation between each variables and GDP: it underestimates the correlation between GDP and consumption, between GDP and hours worked (and in this case is the closest value to the empirical estimate), between GDP and debt repurchases and between GDP and the financial shock, and in this case the size of the underestimation is very large (correlation of 57.71% in the data and of only 0.18% in this simulation).

Table 2.5: **Correlation with real GDP - Sample: 1984:I - 2014.II**

Variables	**Data**	**Original Simulation** J&Q VAR(1) Calibration	**Stochastic Simulation** With VAR(1) Method 1	**Stochastic Simulation** With VAR(2) Method 2
y	1	1	1	1
c	0.9292	0.8210	0.8812	0.8825
n	0.5403	0.8965	0.8843	0.4281
w	0.3254	0.9756	0.9019	0.9169
k	0.2125	0.6530	0.1127	0.7338
b	0.3900	0.6699	0.5330	0.0597
d	0.1149	0.7943	0.6997	0.1038
z	0.1052	0.5660	0.6840	0.9266
ξ	0.5771	0.9269	0.8449	0.0018

2.5.2 Impulse Response Functions

The impulse responses functions (IRF's) reported in this section were all obtained by simulating the model for 3 different cases: a first simulation where the parameters of the model were calibrated with the original Jermann and Quadrini (2012) values, assuming the same VAR(1) system as the authors, for the sample period 1984.I-2010-II; a second simulation for the same economy but using the updated version of the series originally used by Jermann and Quadrini (2012) for the same time period; and a third simulation of the same economy but using the updated version of the series used by Jermann and Quadrini (2012) and for the extended sample period 1984.I-2014.II. For these two last cases, the simulations were first run by assuming a VAR(1) system to characterize the stochastic processes of the two shocks, and then the same simulations were run assuming a VAR(2) system for the same shocks.

Figure 2.28 to Figure 2.31 report the IRF's of output, capital stock, consumption, hours worked, debt, equity payout, and wages to the productivity and the financial shock, both simulated jointly, when the parameters that characterize both shocks were estimated assuming a VAR(1) system, and were updated according to Method 1. Then, Figures 2.32 and 2.31 report the same IRF's for output, capital stock, consumption and hours worked to both shocks, but using the calibrated parameters of both shocks generated by a VAR(2) system and the data were updated according to Method 2.

2.5.2.1 Simulations run assuming a VAR(1) system to calibrate the shocks

Figure 2.28 displays the output and capital stock IRF's to a positive productivity and a positive financial shock, simulated for the three cases explained above. Both output and the capital stock immediately increase after each shock hits the economy, for the three cases considered, but what stands out is the fact both the response of output and the capital stock to the productivity shock in the simulation that uses an updated series to estimate the parameters of both shocks for the extended period 1984.I-2014.II is higher (at least in the first periods after the shock hits the economy) than the simulations that use parameters calibrated according to the original size sample (1984.I-2010.II). This suggests that adding the post-crisis period to the estimation somehow amplificates and propagates further the impact of the productivity shock.

Regarding the financial shock, the responses of output in all cases is positive and even higher than the responses obtained for the productivity shock, but followed immediately by a decreasing path along the simulation horizon. However, the initial response of output in the simulation estimated with the extended sample is lower than the responses of the simulations that use the original sample period. The same is true for the dynamic path of the capital stock when faced with a financial shock, although in this case the capital stock actually increases immediately after the initial impact of the financial shock before eventually fades away.

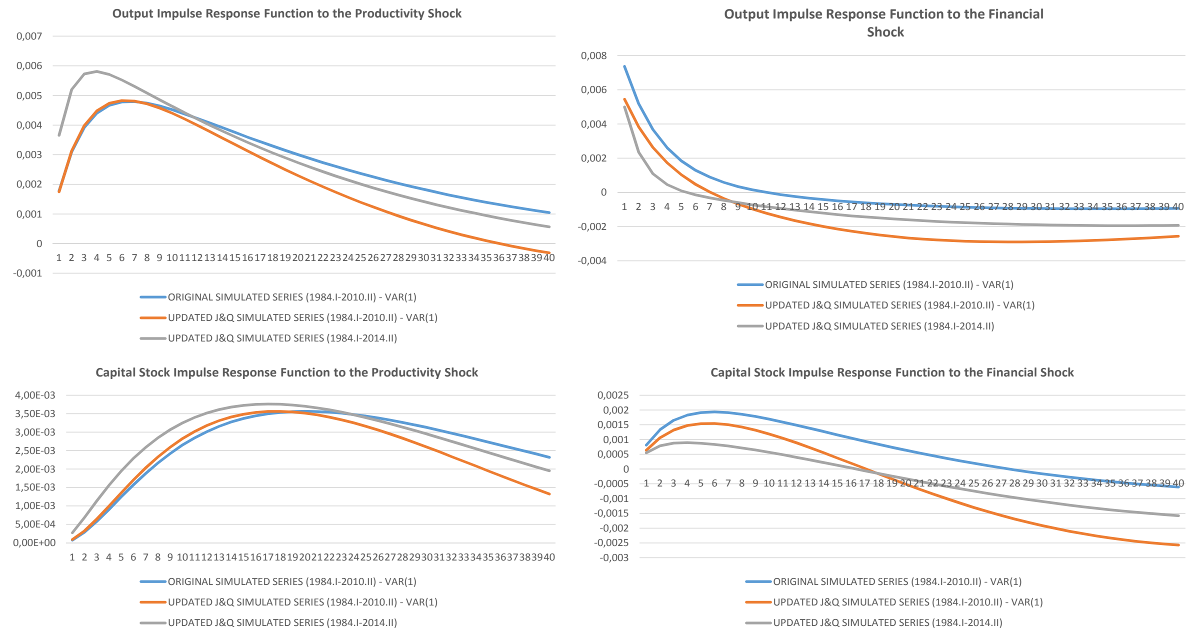


Figure 2.28: Output and Capital IRF's to both Productivity and Financial Shocks

The same behavior can be observed in the IRF's of consumption and hours worked to the productivity and the financial shock, as is illustrated in Figure 2.29. When faced with the productivity shock, both consumption and hours display increasing dynamic paths in the third simulation (estimated with the extended and updated sample (1984.I-2014.II) that are wider and higher than the analogous responses of the other two simulations, at least at the initial impact of the shock. When faced with the financial shock, this pattern changes considerably: the initial response of consumption is negative in all simulations, although it is higher in the third simulation (with the extended sample) than in the other two simulations (with the original data and calibration and with the updated sample but the same time period). However, these paths quickly invert their relative position, because after 6 periods, the consumption IRF in the third simulation is lower than the other two responses, although by period 13 it surpasses the path of the second simulation. In relation to hours, although all the initial responses to the financial shock are positive in the three simulations, the response of the third simulation calibrated with the extended sample size is the lowest of the three.

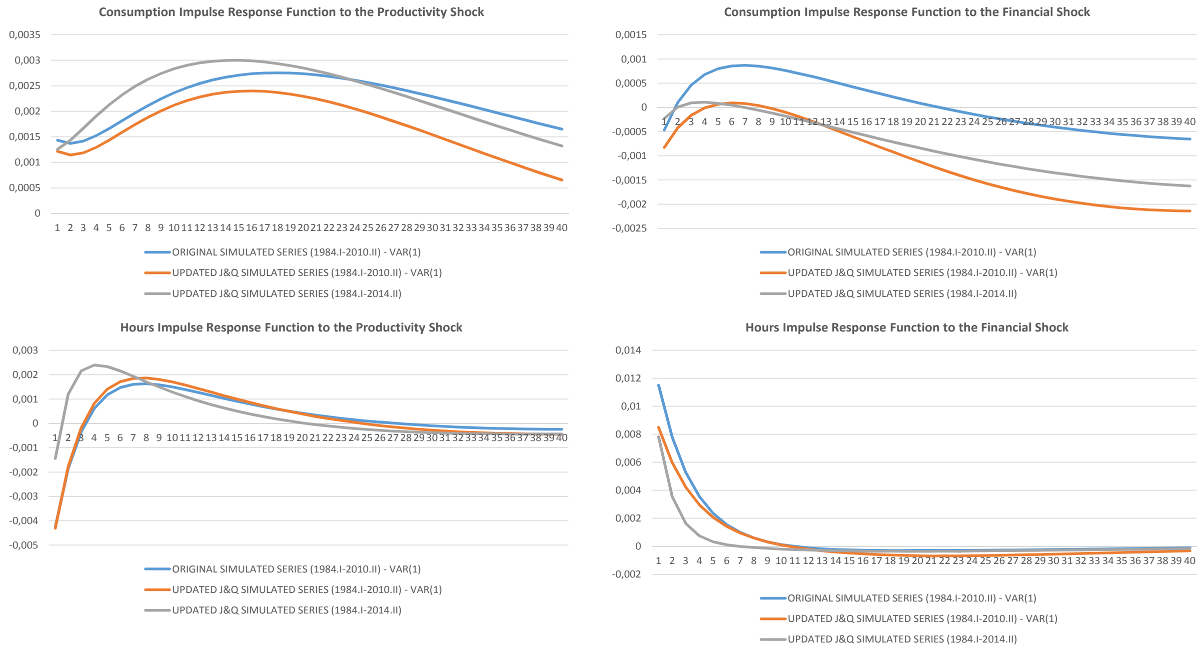


Figure 2.29: Consumption and Hours IRF's to both Productivity and Financial Shocks

Figure 2.30 displays the IRF's of debt and equity payout to a positive productivity shock and a positive financial shock. Regarding total debt, it is clear that all the responses to a positive productivity shock are initially negative, and also that the response of the third simulation is substantially higher than the responses obtained in the other two simulations (that are calibrated using the initial sample period 1984.I-2010.II). Although in all simulations debt increases over time as the effect of the shock fades away, the dynamic path obtained in the third simulation is much smoother than in the other two simulations. In response to a financial shock, debt immediately increases with the impact of the shock in all simulations, but the impulse response is much wider in the first simulation (that uses the original calibration of Jermann and Quadrini (2012)), followed by the second simulation (estimated for the same period but with the updated series), and finally by the lowest and much smoother response, that belongs to the third simulation (estimated with the updated data and for the extended period 1984.I-2014.II).

For the equity payout paid to shareholders, all responses to the productivity shock are negative, which is a puzzling result, since equity payouts tend to generally increase during expansionary periods according to the data, although in all simulations there is a quick recovery and equity payouts start to grow after the initial impact of the shock. In this case, the more extreme response belongs to the third simulation, although it quickly converges to the remaining responses 12 quarters after the shock hits the economy. The opposite behavior is evident when we observe the impulse responses of the equity payout to the financial shock. The initial response is positive for all simulations, but they quickly decay to converge once again to the steady state. However, the highest initial impulse response is obtained in the third simulation, using the extended sample that already includes the subperiod 2010.III-2014.II.

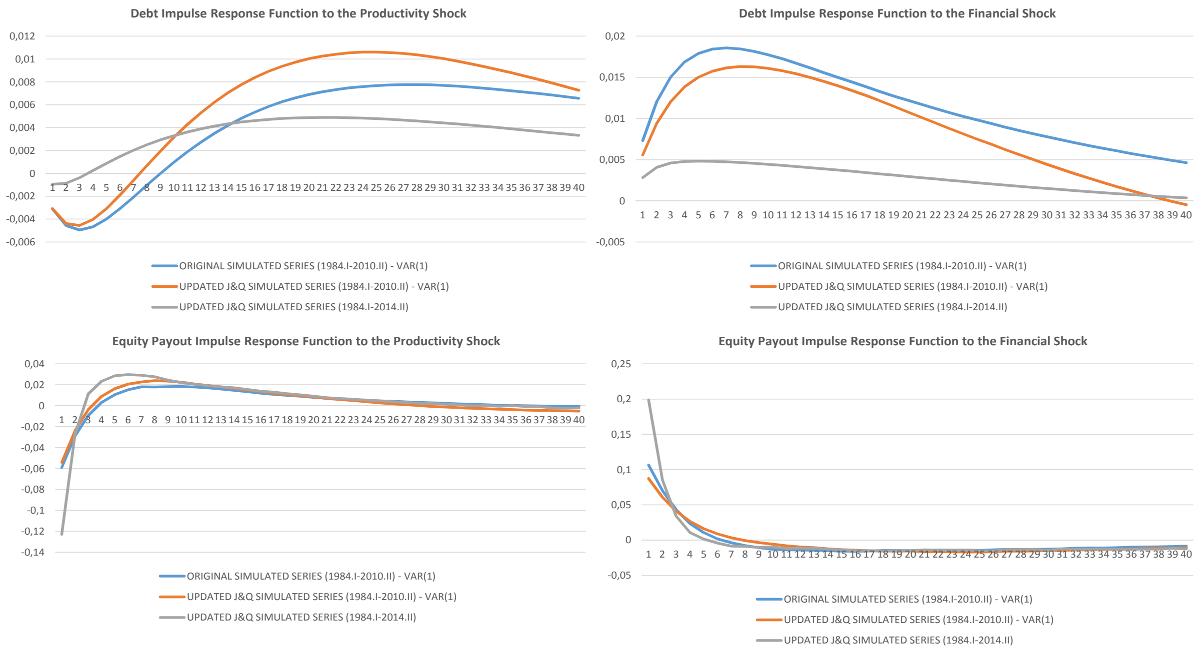


Figure 2.30: Debt and Equity Payout IRF's to both Productivity and Financial Shocks

Finally, in relation to the IRF's of wages to both the productivity and the financial shocks, the pattern observed in the previous cases also repeats here (see Figure 2.31): the wage response to the productivity shock is increasing for all simulations, although the highest path (and the only one that has an immediate positive response to the initial impact of the shock) is the one obtained in the third simulation, using the updated sample for the period 1984.I-2014.II. In relation to the financial shock, the widest wage response belongs to the first simulation, that is estimated with the original Jermann and Quadrini (2012)) calibration. The third simulation initially produces the lowest response, but after 13 periods inverts its position with the response of the second simulation.

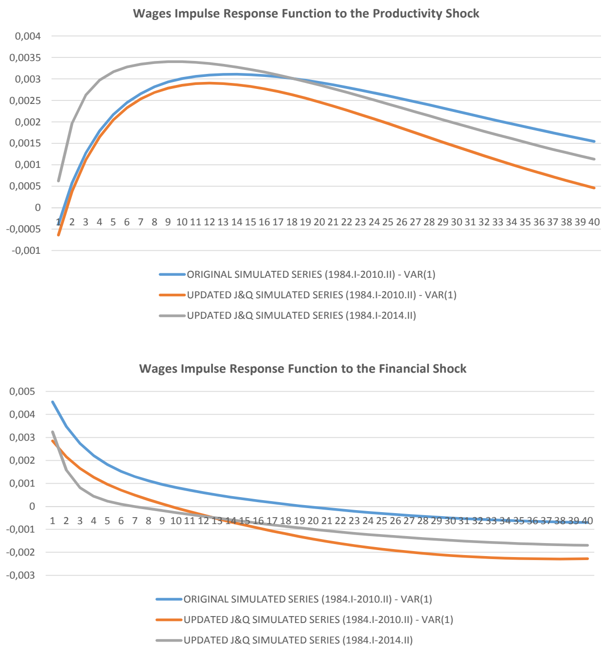


Figure 2.31: Wages IRF's to both Productivity and Financial Shocks

2.5.2.2 Simulations run assuming a VAR(2) system to calibrate the shocks

Figures 2.32 and 2.33 display the impulse response functions for output, capital stock, consumption and hours to the productivity and the financial shocks. As can be observed, using a VAR(2) system to determine the parameters of the two shocks analyzed increase the volatility and amplitude of the impulse responses, since it is a second order approximation to the true parameters that define both shocks, which implies that the nonlinearities proliferate substantially. Furthermore, it is important to take into account that, in this case, the two shocks are not orthogonal, i.e. the correlation of the error terms is definitely not equal to zero, which implies that a shock in one variable is likely to be accompanied by a shock in another variable. However, the findings for these type of simulations are nonetheless presented. It is important to emphasize that, in this case, the second simulation is estimated with the updated sample extended until 2014.II, but the parameters of both shocks were calibrated according to the VAR(1) strategy proposed by Jermann and Quadrini (2012).

As we can observe in Figure 2.32, the impulse responses of output to the productivity shock and to the financial shock computed by assuming a VAR(2) system to characterize both shocks are much broader than the responses obtained in the simulations generated by assuming a VAR(1) process for the same variables. In this case, the output IRF's to the productivity shock are higher for the third simulation (that uses the extended sample until 2014.II) and lower for the first simulation (that uses the original Jermann and Quadrini (2012) calibration), which reinforced the previous result obtained in the analogous analysis conducted for the VAR(1) simulations. This pattern is repeated for the responses of output to the financial shock, but in that case, we get a different conclusion: the highest positive response of output to the financial shock is obtained in the second simulation, and the lowest in the third simulation (although there is an inversion of positions along the dynamic path).

For the capital stock, the impulse responses are even more atypical, specially for the first simulation that uses the original calibration for the period 1984.I-2010.II. The responses of capital to the productivity shock are all positive and increasing for the first and the third simulations, but decreasing in the second simulation. In relation to the financial shock, the responses of capital in the first and second simulations are positive but gradually decrease over the simulation horizon (the highest response is obtained for the first simulation), while in the third simulation that path quickly drops after only 4 quarters, although recovers to the end of the horizon.

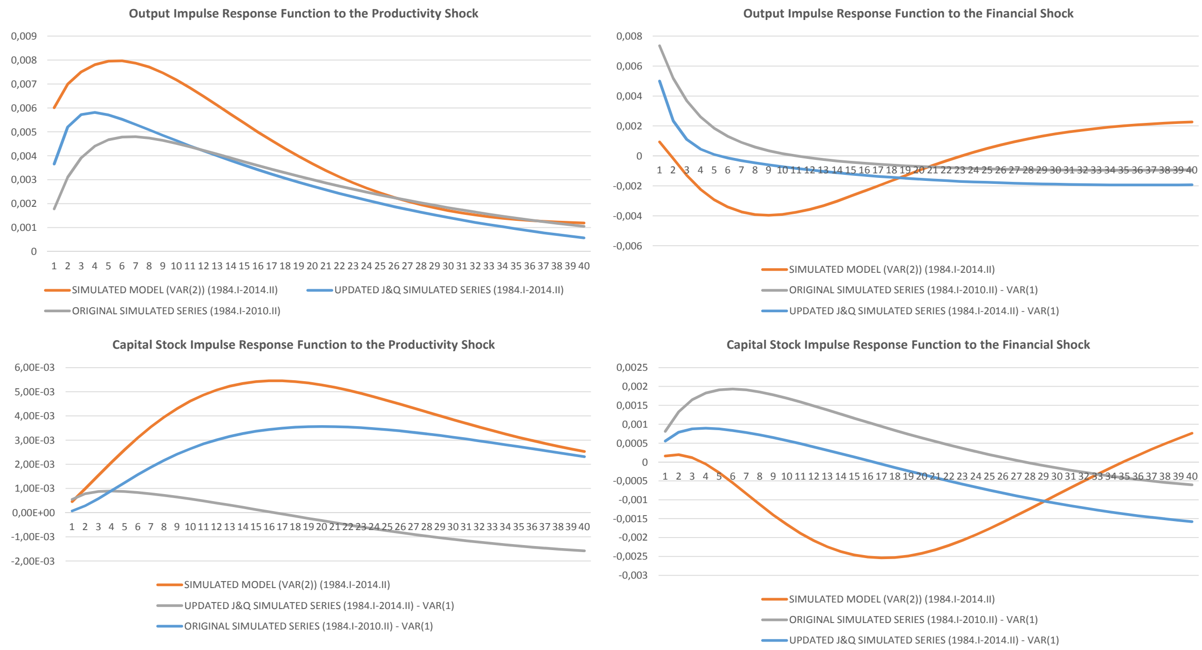


Figure 2.32: Output and Capital IRF's to both Productivity and Financial Shocks

The same atypical behavior is also evident in the IRF's of consumption and hours worked to both shocks, as can be observed in Figure 2.33. In this case, the responses of the first simulation (where the parameters of both shocks were calibrated with the original values proposed by Jermann and Quadrini (2012)) are the lowest and smoother of all simulations, specially in the case of the response of consumption IRF's to the productivity shock. In turn, the third simulation (estimated assuming a VAR(2) system for the shocks and for the period 1984.I-2014.II) presents the more extreme responses to both shocks, although it is somehow more atypical in the of the response of consumption to the financial shock, since it presents a negative and decreasing path after the shock hits the economy.

In relation to the responses of hours worked, the behavior observed assuming a VAR(1) system for the shocks resembles the responses obtained assuming a VAR(2) process: the responses of hours to the productivity shock are increasing, although they are negative at the initial impact of the shock except for the third simulation (estimated assuming the VAR(2) process for the shocks and with the updated extended sample). In relation to the financial shock, the opposite pattern is observed: all the responses of hours are initially positive, but decreasing; and the highest response is generated in the first simulation (estimated with the original sample and calibration for the shocks), while the lowest and smoothest response corresponds to the third simulation (estimated assuming the VAR(2) system for the shocks and the updated sample).

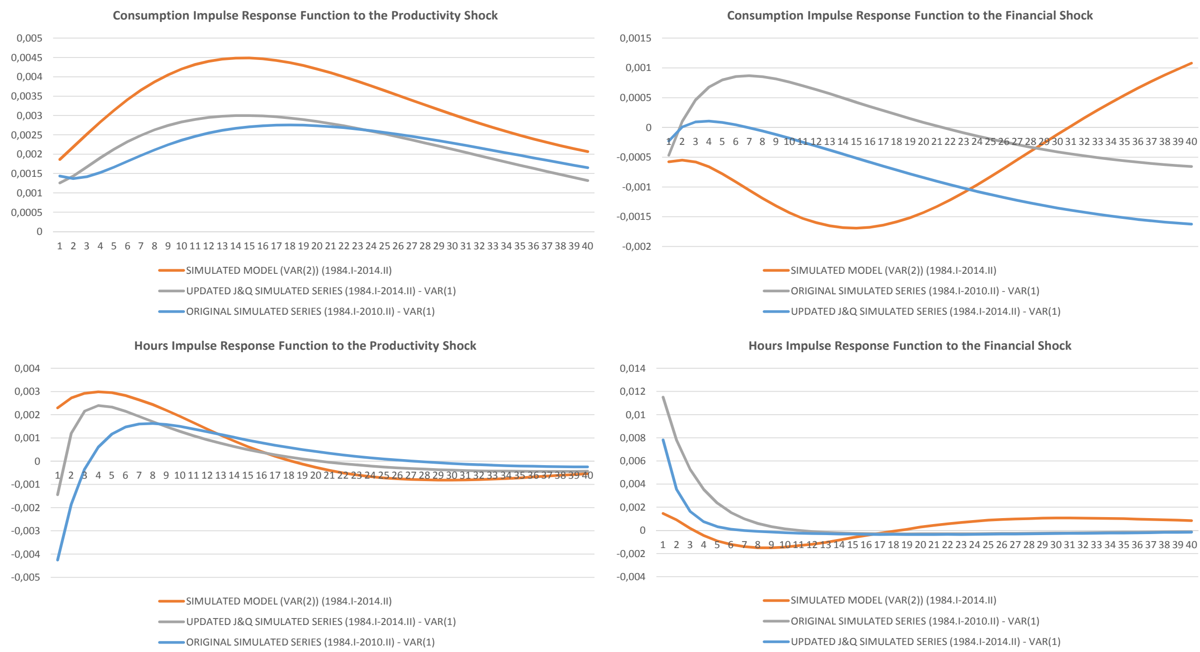


Figure 2.33: Consumption and Hours IRF's to both Productivity and Financial Shocks

2.5.3 Simulated Series

In order to evaluate the performance of the model in generating accurate simulated series of the main macroeconomic and financial variables of the model and compare them against the empirical series of the same variables, I simulated the model assuming a VAR(1) process for both shocks using 3 different approaches:

1. **Deterministic Simulation**: I fed the innovations of the VAR(1) process of the shocks into the model and computed the simulated series for key macroeconomic and financial variables assuming the original Jermann and Quadrini (2012) calibration scheme;

2. **Stochastic Simulation 1**: I calibrated the model with the matrix of the VAR(1) process of the shocks, but not feeding the series of the innovations of both shocks into the model to obtain the simulated series of the main variables of the model, using the original Jermann and Quadrini (2012) calibration scheme;

3. **Stochastic Simulation 2**: I calibrated the model with the matrix of the VAR(1) process of the shocks, but not feeding the series of the innovations of both shocks into the model to obtain the simulated series of the main variables of the model, using the updated and extended sample (1984.I-2014.II) based on the Jermann and Quadrini (2012) calibration scheme.

The comparison between those simulated series and data are displayed in Figures 2.34 to 2.40. At a first glance, in general, these figures suggest that the model is able to replicate reasonably well the behavior of most variables before the 2008 financial crisis (specially in terms of volatility), but it is unable to anticipate or reproduce the behavior of those variables around or during recession periods, specially as deep and lengthy as the 2008 financial crisis and the subsequent Great Recession proved to be.

In relation to output (Figure 2.34), it is possible to observe that none of the simulations is able to replicate accurately the behavior of output observed in the data, specially during the 2008 financial crisis. However, all the simulations approximate the empirical behavior relatively well before the 2008 financial crisis, particularly the two stochastic simulations in the beginning of the simulation horizon (between 1984.I until 1992.IV) and the deterministic simulation 2003.I until 2006.III. From that point on, none of the simulations anticipated the sudden drop that occurred in the second quarter of 2008, neither in depth or length.

The same behavior can be observed for the simulated series of consumption (Figure 2.35), although in this case, the simulations were not able to approximate relatively well the volatility of the empirical series. Before the 2008 financial crisis, specially until the second quarter of 2007, the simulations approximated the empirical series of consumption, but, after this point, they did not anticipated the increase that started during this period that lasted until the end of 2000, and underestimated the empirical path of this variable during this period. When the sudden decrease registered in consumption started to manifest its first signs in the late 2006, none of the simulations anticipated the huge drop that happened later in 2008, although the two stochastic series registered a slight depression between the middle of 2005 until precisely the second quarter of 2008.

For the capital stock (see Figure 2.36), we can observe that the three simulations consistently underestimated the path of capital throughout almost the whole simulation horizon, at least until the third quarter of 2003 (although the simulations seemed to replicate the volatility of the data relatively well). Contrary to the previous variables, the capital stock has been gradually decreasing since the middle of 2000, although it started to decreased at a faster rate at the end of 2008. However, none of the simulations were able to anticipate and replicate such a deep fall, although the stochastic simulations perform slightly better than the deterministic simulation.

In relation to hours worked, we can observe in Figure 2.37 that all simulations fail to anticipate the major downturns and upturns of the empirical series, and even replicate the volatility observed in the data. In this case, it is the deterministic simulation that best approximates the behavior of the empirical series, both in terms of volatility, and in terms of anticipating the moments of booms and recessions. This simulation was even able to anticipate and, although in a lesser degree, reproduce the sudden fall in hours worked in the second quarter of 2008.

An equivalent analysis can be made about the performance of the simulated series for real wages (see Figure 2.38), since none of them was able to replicate the behavior of the empirical series in terms of cyclicality or volatility, specially around and during the 2008 financial crisis. The only period where the simulations were close to the data was between the first quarter of 1989 and the first half of 1996.

In relation to debt, all the simulations were able to perform better and replicate more accurately the trajectory of the empirical series, as we can observe in Figure 2.39, specially in terms of volatility of the series. Although all simulations were able to replicate the debt path between 1984.I and the middle of 1989, from that point on they anticipated (particularly both stochastic simulations) a boom when in reality debt has dropped steadily during this period; and once again in the beginning of 1998 they failed to do so again since both stochastic simulations anticipated a fall when in fact debt has increased during this period. The only series that were consistently closer to the data, both in volatility and behavior, was the series generated by the deterministic simulation, that accompanied relatively well (although sometimes over or underestimating) the trajectory of the empirical series. However, in the second quarter of 2010 this simulation anticipated a sudden recovery of debt after the strong fall caused by the 2008 financial crisis, this is not reflected in the data, where debt continued to decreased until the third quarter of 2013.

Finally, in relation to equity payout (see Figure 2.40), the performance of the three simulations failed to replicate above all the volatility of the empirical series, since all simulates series overestimated strongly that volatility. That difference in volatility is so stark that makes it very difficult to evaluate the performance of the simulated series in terms of the replication of the trajectory followed by the data.

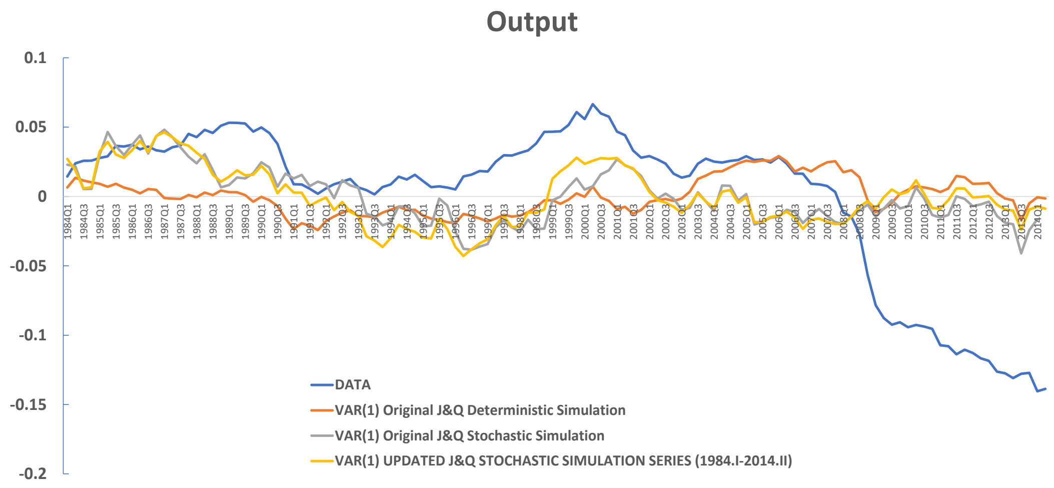


Figure 2.34: Simulated and empirical series for Output

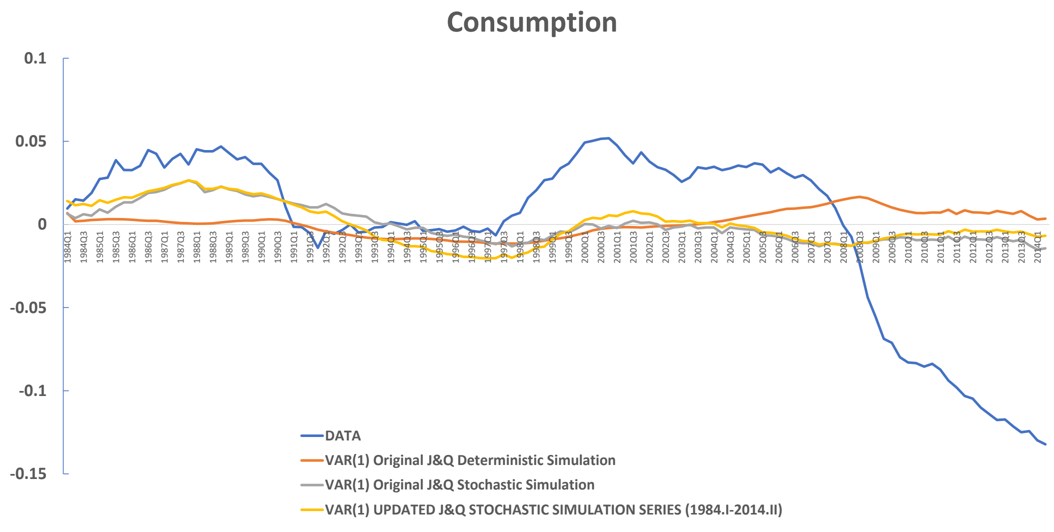


Figure 2.35: Simulated and empirical series for Consumption

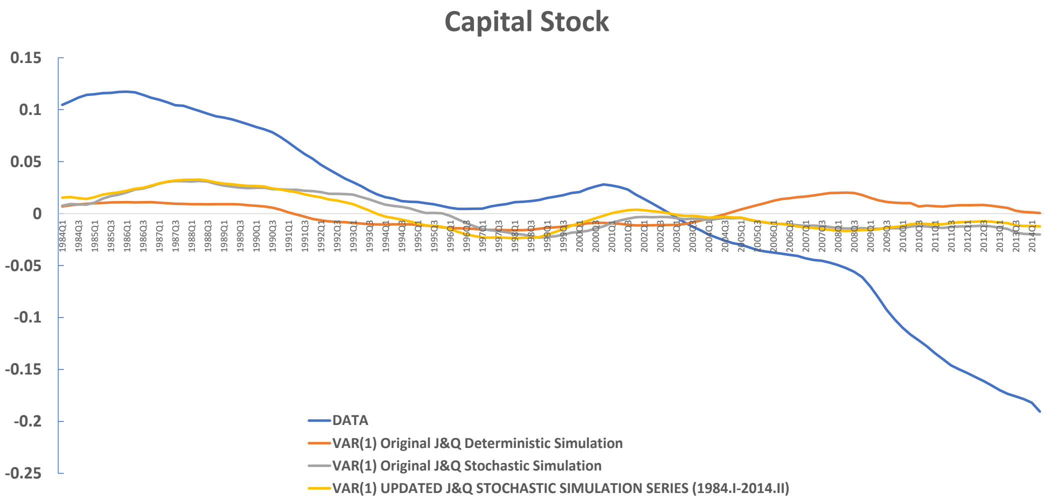


Figure 2.36: Simulated and empirical series for Capital Stock

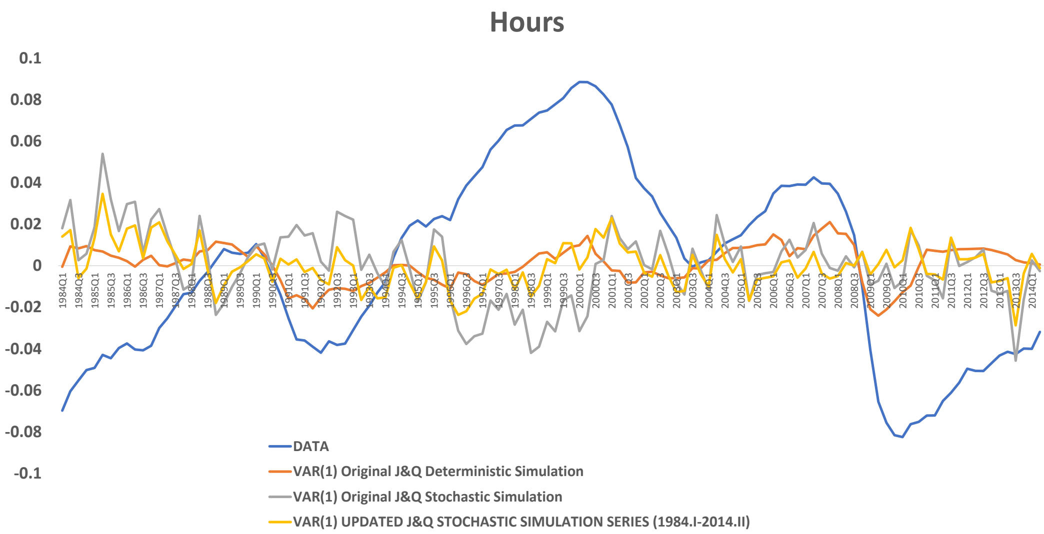


Figure 2.37: Simulated and empirical series for Hours Worked

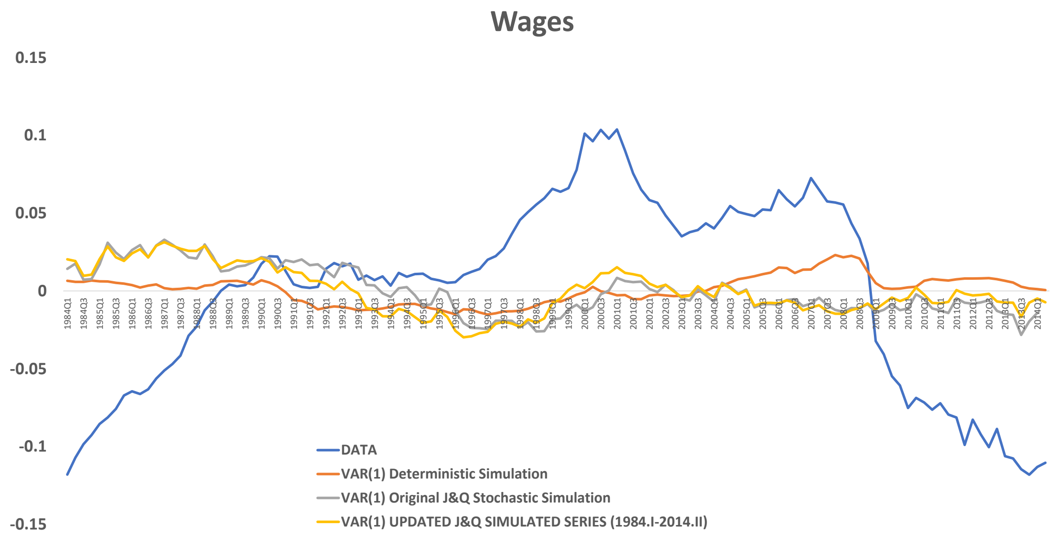


Figure 2.38: Simulated and empirical series for Real Wages

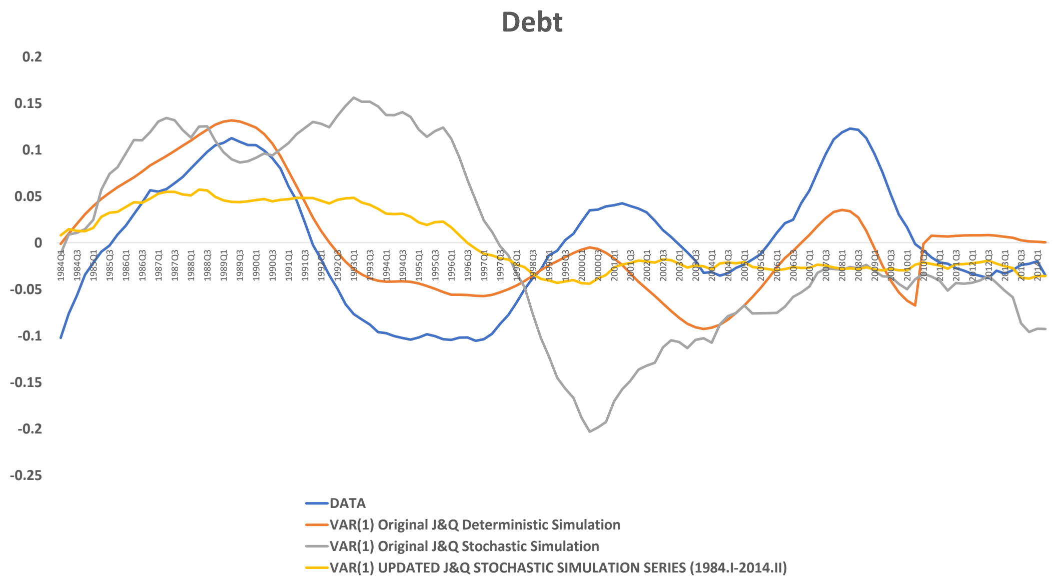


Figure 2.39: Simulated and empirical series for Debt

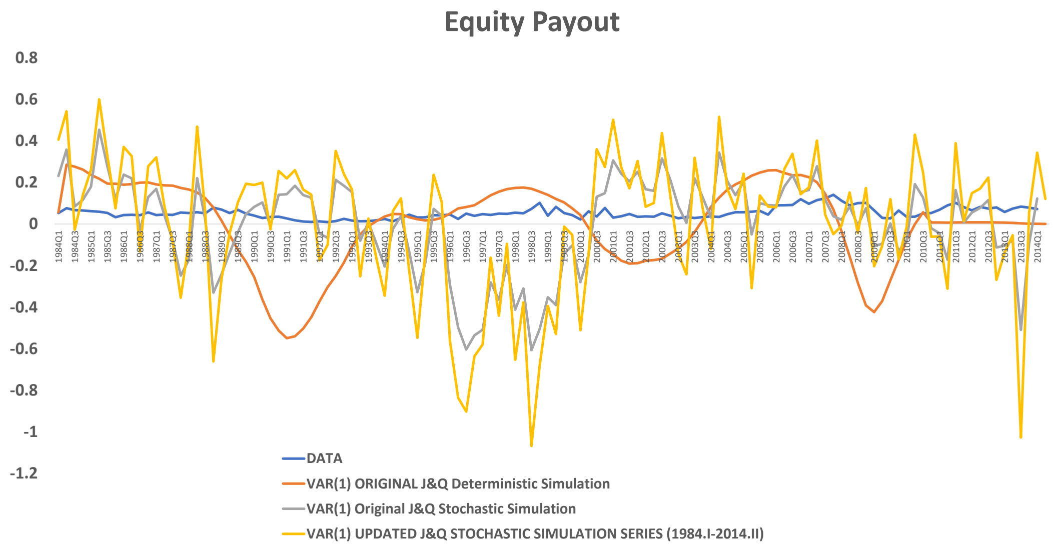


Figure 2.40: Simulated and empirical series for Equity Payout

2.6 Conclusion

In this paper, I compared the empirical time series of major macroeconomic aggregates during the period 1984Q1-2014Q2 with the simulated series computed from the Jermann and Quadrini (2012) model in terms of variability, persistence and amplitude of a productivity and a financial shocks. To conduct that analysis, I compared the respective impulse responses to each shock to the productivity and the financial shocks, the simulated series generated with the model against the data and I also compared the performance of different types of simulations against the data in terms of volatility and correlation with GDP.

In general, in terms of volatility, the model tends to underestimate the volatility observed empirically, except for the simulated series of two shocks (productivity and financial). In terms of correlation with GDP, the model overestimates the data, for almost every variables. However, when considering only the sample 2010.III-2014.II, the results change substantially. Therefore, the 2008 financial crisis period is perceived as a structural break for almost every variable included in the model, both real and financial.

The analysis of the Impulse Response Functions (IRF's) suggest that the inclusion of an updated calibration estimated with an extended sample that includes the subperiod 2010.III-2014.II helps to amplificate and propagate the effects of both the productivity and the financial shock, and also reinforces the idea that the 2008 financial crisis constitutes a structural break in the data for the majority of the variables analyzed.

The simulated series generated by the model, in turn, lead to the conclusion that, although the model is relatively able to replicate and anticipate the trajectory of the data before the 2008 financial crisis, the model is unable to replicate the magnitude and the timing of the huge losses registered by the majority of the real variables during this period.

The 2007-2009 financial crisis constitutes one of the deepest shocks of the last decades, both in terms of propagation and amplification. Overall, the Jermann and

Quadrini (2012) model is able to explain the empirical evidence associated with the time sample considered (1984.I-2010.II). The model seems to replicate relatively well the empirical evidence on volatility and on correlation between the simulated variables and data. However, after updating the data until 2014.II and after re-calibrating the model the model seems unable to reproduce more deep, prolonged and deep recessions triggered by productivity or financial shocks such as the 2008 financial crisis.

Appendix

2.6.1 The Development of the Securitization Process

In order to better understand and explain the securitization process, it is essential to start by clarifying the meaning of certain types of assets, such as collateralized debt obligations (thereby called CDOs). CDOs are structured financial products created by banks to disseminate risk through several investors. In order to create them, banks start by forming diversified portfolios of mortgages and other types of loans, corporate bonds, and other assets like credit card receivables. Then these portfolios are sliced into different types of tranches, which are then sold to investor groups with different risk profiles.

There are essentially three types of tranches: the "super senior tranches", which are considered the safest tranches, since offer investors a (relatively) low interest rate, but it is the first to be paid out of the cash flows of the portfolio; then, in the opposite side of the risk spectrum, there are the "equity tranches", the most junior tranches or "toxic waste", which are paid only after all other tranches have been paid out; and the mezzanine tranches, between these two extremes.

The exact cutoffs between the tranches are typically chosen to ensure a specific rating for each tranche. For example, the top tranches are constructed to receive a AAA rating. The more senior tranches are then sold to various investors, while the toxic waste is usually (but not always) held by the issuing bank, to ensure that it adequately monitors the loans. Buyers of these tranches or regular bonds can also protect themselves by purchasing credit default swaps (CDS), which are contracts insuring against the default of a particular bond or tranche. The buyer of these contracts pays a periodic fixed fee in exchange for a contingent payment in the event of credit default. Estimates of the gross notional amount of outstanding credit default swaps in 2007 range from \$45 trillion to \$62 trillion. One can also directly trade indices that consist of portfolios of credit default swaps, such as the CDX in the United States or iTraxx in Europe. Anyone who purchased a AAA-rated tranche of a collateralized debt obligation, combined with a credit default swap, had reason to believe that the investment had low risk, because the probability of the CDS counterparty defaulting was considered to be small.

Most investors prefer assets with short maturities, such as short-term money market funds. It allows them to withdraw funds at short notice to accommodate their own funding needs (for example, Diamond and Dybvig (1983); Allen and Gale (2007)) or it can serve as a commitment device to discipline banks with the threat of possible withdrawals (as in Calomiris and Kahn (1991); Diamond and Rajan (2001)). Funds might also opt for short-term financing to signal their confidence in their ability to perform (Stein, 2005). On the other hand, most investment projects and mortgages have maturities measured in years or even decades. In the traditional banking model, commercial banks financed these loans with deposits that could be withdrawn at short notice.

The same maturity mismatch was transferred to a shadow banking system consisting of off-balance-sheet investment vehicles and conduits. These structured investment vehicles raise funds by selling short-term asset-backed commercial paper with an average maturity of 90 days and medium-term notes with an average maturity of just over one year, primarily to money market funds. The short-term assets are called asset backed because they are backed by a pool of mortgages or other loans as collateral. In the case of default, owners of the asset-backed commercial paper have the power to seize and sell the underlying collateral assets. Asset-backed commercial paper had become the dominant form of outstanding commercial paper by the start of 2006.

The strategy of off-balance-sheet vehiclesinvesting in long-term assets and borrowing with short-term paper exposes the banks to funding liquidity risk: Investors might suddenly stop buying asset-backed commercial paper, preventing these vehicles from rolling over their short-term debt. To ensure funding liquidity for the vehicle, the sponsoring bank grants a credit line to the vehicles, called a liquidity backstop. As a result, the banking system still bears the liquidity risk from holding long-term assets and making short-term loans even though it does not appear on the banks' balance sheets.

Another important trend was that the maturity mismatch on the balance sheet of investment banks increased. This change was the result of a move towards financing balance sheets with short-term repurchase agreements, or repos. In a repo contract, a firm borrows funds by selling a collateral asset today and promising to repurchase it at a later date. The growth in repo financing as a fraction of investment banks' total assets is mostly due to an increase in overnight repos. The fraction of total investment bank assets were financed by overnight repos roughly doubled from 2000 to 2007. Term repos with a maturity of up to three months have stayed roughly constant at as a fraction of total assets. This greater reliance on overnight financing required investment banks to roll over a large part of their funding on a daily basis.

In summary, leading up to the crisis, commercial and investment banks were heavily exposed to maturity mismatch both through granting liquidity backstops to their off-balance sheet vehicles and through their increased reliance on repo financing. Any reduction in funding liquidity could thus lead to significant stress for the financial system, as we witnessed starting in the summer of 2007.

Another important aspect of the securitization process was the increasing widespread of several structured products that allow to satisfy the different risk needs of different investors groups. Risk can be shifted From the investors who are more risk averse to those who wish to bear it, and it can be widely spread among many market participants, allowing for lower mortgage rates and lower interest rates on several types of loans. One of the most important innovations introduced with the securitization process was the possibility that certain institutional investors to hold indirectly off-the-balance-sheet assets that they were not allowed to hold previously by mandatory requirements. The implementation of important changes in the regulatory and ratings arbitrage was one of the forces behind the increasing demand for structured investment vehicles, during the pre-2007 crisis period.

For example, some international agreements on bank regulation were implemented, such as Basel I and Basel II accords, which imposed important rules on capital requirements of banks (designated as "capital charges"). The Basel I accord required that banks hold capital of at least 8 percent of the loans on their balance sheets, but this rule could be easily overcome if banks move a pool of loans into off-balance-sheet vehicles, and then granting a credit line to that pool to ensure a AAA-rating, allowing banks to reduce the amount of capital they needed to hold to satisfy Basel I regulations without changing the amount of risk for the bank. The subsequent Basel II accord implemented capital charges based on asset ratings in order to correct this problem, but banks were able to reduce their capital charges by pooling loans in off-balance-sheet vehicles, therefore reducing the idiosyncratic risk for each bank. As a consequence, assets issued by these vehicles received a better rating than did the individual securities in the pool. In fact, issuing short-term assets improved the overall rating even further, since banks sponsoring these structured investment vehicles were not sufficiently downgraded by rating agencies for granting liquidity backstops. In addition, securitized assets may have received more favorable ratings compared to corporate bonds because rating agencies (especially Moody's, Standard & Poor's and Fitch) collected higher fees for structured products.

The major consequence of this increased demand for securitized, short-term products, which could be moved out of the balance sheet in order to maintain the risk credit borne by banks, ultimately led to a boom of cheap, unrestricted and unsurveilled credit in the banking markets, leading to very low lending standards. This combination of factors, in turn, resulted in the housing bubble that escalated during the 1990's until 2007, that was in the root of the financial crisis.

Chapter 3

Using Tax Subsidies to Overcome the Zero Lower Bound

3.1 Introduction

This paper takes a medium-scale New Keynesian DSGE model proposed by Jermann and Quadrini (2012) (based on the model originally developed by Smets and Wouters (2007)) [1] to investigate the role of a subsidy on the interest rate as an effective fiscal policy tool capable of overcoming a liquidity trap. In this case, the liquidity trap takes the form of a zero lower bound constraint on the nominal interest rate. Using the Jermann and Quadrini (2012) model as the primary theoretical reference to conduct this analysis allows us to study the lasting post-2008 financial crisis period characterized by low economic growth, low inflation and near zero interest rates.

Since the 2008 financial crisis, now designated as the Great Recession, the major

[1]The model by Jermann and Quadrini (2012) was initially designed to analyse the macroeconomic effects of financial shocks. Jermann and Quadrini (2012) have shown that financial shocks had one of the most substantial contributions to the volatility of the growth rate of output in the aftermath of the 2008 financial crisis (approximately 46%). The authors also showed that financial shocks can capture almost all of the decline in GDP in the third quarter of 2008 and about half of the decrease in the fourth quarter of 2009.

economies such as the U.S. have experienced an unusual combination of features that include the weak growth of the GDP, low core inflation, and a real interest rate close to zero. The process of recovery has been much slower than usual in most high-income countries and has lasted longer than in previous financial crises, which suggests that other mechanisms are at play, in addition to the central banks' efforts to stabilize the financial system through quantitative easing measures. The Federal Reserve (Fed) pushed the federal funds rate to near zero by the end of 2008, and held it there through 2011 and beyond, which allowed commercial banks to accumulate substantial cash reserves in their accounts. This pattern of excess liquidity in the market accompanied by a real interest rate close to zero implies the presence of a liquidity trap, under which conventional central bank open-market operations are exceedingly ineffective as an expansionary policy tool. Monetary authorities, therefore, began to consider alternative policy approaches for escaping the trap, including raising the inflation target, depreciating the currency and targeting long-term interest rates directly. However, less conventional policies that involve fiscal measures are also considered, such as an excess reserve tax for commercial banks and a major expansion in the federal loan guarantee program for smaller businesses, both studied by Pollin (2012).

In this paper I study the role of a tax benefit on the interest rate as an instrument to overcome a liquidity trap. I also analyze whether the imposition of a zero lower bound constraint amplifies or not the negative impact of a financial shock in the economy, particularly during and after the 2008 financial crisis. To accomplish this goal, I use the structural DSGE model developed by Jermann and Quadrini (2012), originally estimated with Bayesian maximum likelihood techniques. This model was constructed as a second approach of a DSGE model aimed to explore how the dynamics of real and financial variables are affected by financial shocks and financial frictions. Their first approach was a parsimonious real business cycle (RBC) model with only two shocks (the standard technological shock and the financial shock), and this second method was developed to assess the contribution of the financial shock in comparison with seven other shocks, thus providing a richer and more robust scenario of the impact of financial shocks on the economy. This structural approach follows closely the model estimated by Smets and Wouters (2007), but with the addition of financial frictions and financial shocks. Then, I compare the main findings against the results obtained in the most recent related literature, particularly the model estimated by Lindé et al. (2016). These authors update the Smets and Wouters (2007) by adding a financial accelerator mechanism, shocks originated in the financial sector and by introducing explicitly the zero lower bound into the model. They extend the model with a financial accelerator and allow for time-variation in the endogenous propagation of financial shocks. However, their approach to the financial shock and to the financial friction differ significantly from the approach presented by Jermann and Quadrini (2012), and they do not focus on the role of the interest rate subsidy as an attainable instrument to circumvent the effects of the zero lower bound, which is the fundamental premise of this paper.

In this paper I incorporate the corrections and changes proposed by Pfeifer (2016), who showed that the Jermann and Quadrini (2012) model suffers from several methodological issues which affect considerably some of the findings of the model. Pfeifer (2016) showed that the TFP series constructed by Jermann and Quadrini (2012) combined the GDP of the total economy with inputs (capital stock, depreciation, debt stock) related to the private business sector only. The author also remarked that the structural framework was affected by some errors in three equilibrium conditions and that some of the steps undertaken to perform the Bayesian estimation also suffered from some issues. After fixing these problems and reestimating the model, Pfeifer (2016) concluded that the marginal efficiency of investment (MEI) shocks are the largest contributors to generate output volatility, and that financial shocks, in contrast, accounted for only 6.5% of output variance as opposed to the 46% originally reported by Jermann and Quadrini (2012). Despite this finding, Pfeifer (2016) acknowledged that financial shocks contribute with approximately 2 to 3 percentage points to the observed GDP decline during the Great Recession, ensuring that studying financial shocks as a source of volatility in the economy remains relevant.

The introduction of the zero lower bound constraint into this model share some similarities with several models well established in the literature, such as Correia et al. (2013), Carlin and Soskice (2018), Chari et al. (1991), Christiano et al. (2011), Eggertsson (2009), Eggertsson and Woodford (2003), Eggertsson and Woodford (2006) and Krugman (1998). The majority of these models are built upon New Keynesian foundations and were constructed to characterize the monetary policy at the zero lower bound, but also to study the beneficial effects that a well designed fiscal policy can provide by improving total welfare. Although these models study different set-ups for the fiscal and monetary policies (jointly or independently) and analyse the role of different instruments (such as lump sum taxes, wage and consumption taxes, sales taxes and investment tax credits, etc) to overcome the zero lower bound problem, and assuming the presence of different types of frictions (price and wage stickiness, adjustment costs of capital, collateral constraints, financial intermediation, heterogeneous agents, etc), the literature is absent in explaining the role of interest rate subsidies in circumventing a liquidity trap.

Furthermore, the majority of these models are designed and calibrated to reflect the pre-crisis period and are not fully equipped to accommodate structural breaks in estimated coefficients, moments and probabilistic density of those models during the peak of the recession and the post-crisis slow recovery period. For instance, the Smets and Wouters (2007) framework, which includes a large number of shocks, was adopted as the standard version of several models used by central banks and several authors to study the economy in the pre-2008 era and conduct forecasting exercises. However, after the Great Recession of 2008, many researchers realized that to account for the depth of the recession in a more efficient capacity, this model needed additional features such as unlikely shocks and financial frictions that could explain the slow recovery and the missing disinflation following the great recession. Hence, there is a vast new branch of the literature that extend the New Keynesian founded structural macroeconomic models as the benchmark by adding different types of shocks and frictions to the original setup. One of those cases is the model developed by Lindé et al. (2016), which augments the original Smets and Wouters (2003) and also the Smets and Wouters (2007) structure through three extensions: first, they take the zero lower bound explicitly into account when estimating the model over the full sample [2]; second, they introduce unexpected shocks such as a risk-premium and investment-specific technology shocks to the benchmark model and to show that these shocks are non-Gaussian they also add regime switching processes in the volatility of some of those shocks; and third, they conduct an exhaustive analysis to understand how the performance and properties of the basic model can be improved by introducing a financial accelerator mechanism and shocks originated in the financial sector. [3] However, they conclude that current extensions of the benchmark model (at least as they addressed them) are often not rich enough to analyze the major policy challenges that both the monetary and macroprudential policy have to face in the current global macroeconomic environment.

The main goal of the implementation of the zero lower bound constraint in the context of the New Keynesian structural model proposed by Jermann and Quadrini (2012) is to analyse how a subsidy over the nominal interest rate can be used to entirely avoid the ZLB restriction and replicate the effects of a economy where the nominal interest rate can float freely. The methodology adopted to implement the ZLB constraint follows closely the procedure conducted by Correia et al. (2013), in which they introduce the ZLB restraint assuming that the monetary policy follows a Taylor rule truncated at zero. They show how a combination of different distortionary taxes and other fiscal instruments (designated as unconventional fiscal policy) [4] is able to neutralize the effects of the zero lower bound constraint, and enables the implementation of the first-best allocation, depending on the set of available instruments and on the assumptions adopted in the model (such as inclusion of capital and stickiness of prices and wages). Other researchers such as

[2] Lindé et al. (2016) use two alternative approaches to accomplish the introduction of the ZLB into the model: first, they "implement the ZLB as a binding constraint on the policy rule with an expected duration that is determined endogenously by the model in each period", and second, the authors " impose the expected duration of the ZLB spells during the recession to be consistent with external information derived from overnight index swap rates"

[3] Lindé et al. (2016) insert an additional observable variable, the Baa-Aaa corporate credit spread, to engineer the financial accelerator, based upon the Bernanke et al. (1999).

[4] The scheme proposed by Correia et al. (2013) combines an "increasing path for consumption taxes and a decreasing path for labor taxes", and it also entails the adoption of temporary measures such as an investment tax credit and a cut in capital income taxes.

Christiano et al. (2011), Eggertsson (2009), Fernández-Villaverde et al. (2015) and more recent works such as Nakata (2017) also use the truncated Taylor rule at zero to incorporate the ZLB constraint into the model.

In this paper I will focus on the interest rate subsidy as the only fiscal policy instrument, besides lump-sum taxes. Other papers that have studied the interaction between the effects of financial shocks and the adoption of non-standard monetary policy instruments when the zero lower bound is binding are Eggertsson and Woodford (2006), Correia et al. (2013) and also Correia et al. (2016), that study how the optimal monetary and fiscal policy can be used in a monetary model with financial intermediaries and conclude that credit subsidies can be employed to overcome the zero lower bound constraint on the nominal interest rate, therefore shielding the economy from financial shocks on credit spreads.

The structural approach adopted by Jermann and Quadrini (2012) follows closely the model initially constructed by Smets and Wouters (2007), which includes seven shocks (productivity, marginal efficiency of investment (MEI), intertemporal preferences, labor supply, price markup, government spending and monetary policy) to which Jermann and Quadrini (2012) added the financial shock and the financial frictions. The contribution of this paper is to use this extended model to infer if the introduction of the interest rate subsidy is enough to circumvent the zero lower bound constraint and escape a liquidity trap and to measure how the presence of each one of these shocks can stimulate or hinder the achievement of that objective.

Although this model includes eight shocks, there is a smaller subset of shocks (e.g. productivity, financial or preference shocks) that are generally more popular when performing simulations of the model since they usually generate larger responses from the endogenous variables, and consequently, increase the probability that the ZLB constraint will bind. Jermann and Quadrini (2012) justified the importance of both financial and productivity shocks as key forces driving the large fluctuations in the U.S. economy in the Great Recession, and the inclusion of the preference shock is also pivotal because Correia et al. (2013) argue that, based on the findings of Christiano et al. (2011) and Eggertsson (2009), the interaction of this shock with the zero lower bound can generate a potentially big recession. Furthermore, according to Fernández-Villaverde et al. (2015) "positive shocks to the discount factor or to the productivity raise the probability of the economy hitting the ZLB." Williamson (2017) also argues that a temporarily-high discount factor shock or a temporarily-low productivity growth shock help to induce a binding ZLB constraint, where inflation and output are higher than they would otherwise be. In this paper, all eight shocks are used to simulate the model and study the impact of the ZLB constraint. However, in this case the more conventional shocks (preference, technology and government expenditures shocks) and even the financial shock have a much lower contribution to the volatility of output, labor and consumption and investment than in the original Jermann and Quadrini (2012) framework (largely due to the reestimation conducted by Pfeifer (2016)). Furthermore, the impulse responses of some variables to these shocks (particularly investment) do not fit completely with the expected outcomes predicted by economic intuition, and that is the reason why the simulation exercises undertaken in this paper also focus in the role of some less explored shocks in the zero lower bound literature, such as the marginal efficiency of investment (MEI) shock or the price markup shock.

In order to impose the zero lower bound constraint into the Jermann and Quadrini (2012) model, I simulate the model using Occbin, a library of numerical routines developed by Guerrieri and Iacoviello (2015) that solves the model through a piecewise linear solution algorithm compatible with Dynare. This model includes only one occasionally binding constraint, which implies that the nominal interest rate cannot be lower than zero (the zero lower bound constraint, henceforth the ZLB constraint). In such cases, the Occbin algorithm is designed to assume the existence of two regimes: one where the occasionally binding constraint is slack and another one where the constraint is binding. The model is linearized under each regime around the non-stochastic steady state, and choose the regime that applies at the point of linearization as the "reference" regime and the other regime as the "alternative" [5]. It is important to stress that the Occbin algorithm demands the fulfilment

[5] Guerrieri and Iacoviello (2015) stress that it is irrelevant whether the occasionally binding constraint is binding at the reference regime or the alternative regime.

of two requirements to ensure its correct implementation: the reference regime must ensure that the conditions for existence of a rational expectations solution recommended by Blanchard and Kahn (1980) hold; and that "if shocks move the model away from the reference regime to the alternative regime, the model will return to the reference regime in finite time under the assumption that agents expect that no future shocks will occur".

This paper is organized as follows: in section 2, a brief discussion of the literature related to the zero lower bound and financial shocks and financial frictions is presented. In Section 3 the full New Keynesian DSGE model developed by Jermann and Quadrini (2012) is explained. Then, I simulate the fiscal policy with the interest rate subsidy at the ZLB in Section 4 and discuss the implications of my results on the evaluation of this type of fiscal instrument. Section 5 concludes.

3.2 Literature

This paper builds upon extensive literature on the zero lower bound in New Keynesian DSGE models, although not all of those studies include financial shocks and financial frictions. Most of those models focus primarily on studying the optimality of monetary policy and fiscal policy at the zero lower bound, where the government and the monetary authority use the nominal interest rate and a wide range of different taxes and subsidies as policy instruments. Relative to this paper, authors such as Krugman (1998), Correia et al. (2013), Carlin and Soskice (2018), Chari et al. (1991), Christiano et al. (2011), Eggertsson (2009), Eggertsson and Woodford (2003), Eggertsson and Woodford (2006) and many others analyse a much more wide menu of fiscal instruments, which include direct and distortionary taxes (on savings, capital, consumption, wages, etc) and subsidies (on credit, on investment, etc). Despite many of these authors give more attention to the ability of the government or the central bank to combat deflation or to overcome distortions imposed by staggered prices or staggered information, there is a consensus that the zero bound represents a severe challenge for policymakers, therefore justifying the imposition of inefficient policies or the use of unconventional tools, as Correia et al. (2013) suggests.

Although that there are several studies on the zero lower bound investigating alternative instruments of both monetary and fiscal policies, the role of an interest rate subsidy as a fiscal policy tool to overcome the zero lower bound constraint is mostly missing in the literature. The two closest studies to this theme are Correia et al. (2016) and Singh (2014), although they use very different approaches to characterize their models and to define the interest rate subsidy.

Correia et al. (2016) use a broader set of monetary and fiscal policy instruments to study optimal policy in a monetary model with costly financial intermediation. In this model, banks and other financial intermediaries are "subject to an enforcement constraint that generates inefficiently high and volatile credit spreads." Under the zero lower bound constraint, the manipulation of the nominal interest rate as the standard instrument of monetary policy is limited. Therefore, to escape this constraint, central banks consider alternative tools, which include different forms of credit policies. The authors focus mainly in the role of credit subsidies as a useful tool to overcome liquidity traps and show that, under certain assumptions, credit subsidies can be employed to shelter the economy from the adverse effects of financial shocks on credit spreads. Furthermore, they argue that credit subsidies are particularly useful to assist the interest rate policy in avoiding other distortions, especially those associated with the stickiness of prices and information. The principal conclusion of this paper is that credit subsidies are the best choice to shield the economy from the adverse consequences of financial shocks on credit spreads. However, according to Correia et al. (2016), standard tax instruments are sufficient to overcome the zero lower bound constraint on interest rates, which is an encouraging argument to support the utilization of an interest rate subsidy as a policy tool capable of alleviating the effects of the zero lower bound constraint.

The paper by Singh (2014) shows that a tax-subsidy scheme can be used to overcome the zero lower bound on interest rate, without any inflation manipulation or inflationary expectations. However, the author uses a simple Old Keynesian model instead of a New Keynesian DSGE version, which is a standard closed economy macroeconomic model whose equilibrium can be synthetized in five equations: an identity equation that equates savings to investment in equilibrium; an equilibrium condition equating the supply and the demand of money; the production function that relates output with employment (in the version without capital); a condition that states that in equilibrium the real wage is equal to the marginal product of labor; and an equilibrium condition to ensure that the supply of labor depends on real wage. Although Singh (2014) follows closely Correia et al. (2013) to show that fiscal policy can be used as a credible alternative to monetary policy in circumventing the zero lower bound constraint, Singh (2014) argues that the approach developed by Correia et al. (2013) avoids the ZLB problem by manipulating inflation through fiscal policy, while Singh (2014) avoids using inflation altogether. Instead the author uses a tax-subsidy scheme policy implemented by the treasury that attempt to mimic the tax-subsidy scheme implicit in inflation, which is controlled by the central bank. This scheme consists on an explicit tax rate on savings counterbalanced by the presence of an explicit subsidy rate on investment. However, this subsidy rate on investment assumes a lump-sum form, contrary to the distortionary definition of the interest rate subsidy studied in this paper.

Recently, a lot of effort have been devoted to incorporating the financial sector into more traditional DSGE models. Therefore, integrating the analysis of financial markets explicitly into general equilibrium is a top priority in the literature, both for firms and households. Financial frictions and financial shocks such as an equity premium shocks are especially important to capture and evaluate different phenomena generated in financial markets, such as the impact of credit supply conditions, risk aversion, anticipations about future policy actions or the effect of quantitative easing (QE) policies targeting the yield curve or risk spreads. Other authors such as Iacoviello (2005) and Guerrieri and Iacoviello (2017) incorporate borrowing and collateral constraints on housing wealth to drive asymmetries in the link between housing prices and economic activity, while other models (Christiano et al. (2003, 2007, 2010), Gertler and Kiyotaki (2010), Gertler and Karadi (2011)) introduce an active role for financial institutions (traditional banks, investment banks, hedge funds, etc) in the credit supply process or the asset pricing functions. There is also a new branch of macro-financial models (He and Krishnamurthy (2012), Allen and Gale (2004), Adrian and Shin (2010a), Brunnermeier and Sannikov (2014), Chen and Song (2009), Mendoza (2010)) that explore in more depth how constraints over financial intermediaries can affect their role in setting asset prices and influencing the endogenous risk that investors can support, therefore disrupting the channels of transmission of shocks between the financial and the real sectors that can help to explain patterns during financial crises. Another important reference related to the financial frictions branch of literature is Negro et al. (2017), that incorporate the credit frictions introduced by Kiyotaki and Moore (2012) in a relatively standard dynamic stochastic general equilibrium (DSGE) model [6] that features the usual nominal and real frictions in this type of models, such as price and wage rigidities à la Calvo and aggregate capital adjustment costs. In this setting, monetary policy is also implemented through manipulations of the nominal interest rate according to a standard Taylor rule truncated by the zero lower bound. Contrary to the Jermann and Quadrini (2012) case, the credit disturbances proposed by Kiyotaki and Moore (2012) can be divided into two different types: financial constraints (that, like collateral constraints, arise when a firm can borrow only up to a fraction of the value of its current investment) and liquidity constraints (that arise when a firm, when faced with an investment opportunity, can sell only up to a certain fraction of the more less liquid assets [7] on its balance sheet in each period. They conclude that both the financial shock and the liquidity policy can have a quantitatively large effect, and a binding zero lower bond plays a crucial role to achieve those findings.

[6]Following closely the framework proposed by Christiano et al. (2005) and Smets and Wouters (2007).

[7]These illiquid assets correspond to equity holdings of other firms, which include privately issued paper such as commercial paper, bank loans, mortgages, etc.

3.3 General Overview of the Model

In this paper I follow closely the New Keynesian DSGE model estimated by Jermann and Quadrini (2012), which was constructed as an extended structural approach of their real business cycle model destined to study the macroeconomic effects of financial shocks over the economy, in the aftermath of the 2008 financial crisis. The main difference between the two approaches (in addition to the usual standard features that differentiate both models) lies in the fact that the real business cycle framework is independent of how many shocks are added to the model, whereas in the structural estimation the effects of a particular shock depend, in general, on the shocks that are included in the model. This structural approach follows closely the model estimated by Smets and Wouters (2007), but extends the original model by adding financial frictions and financial shocks. This framework includes the original seven shocks assumed by Smets and Wouters (2007) (productivity, investment-specific, intertemporal preferences, labor supply, price markup, government spending and monetary policy) but it also includes an additional financial shock and financial frictions to expand the scope of the model. Eight empirical variables were used to estimate the model: GDP, investment, working hours, wage rate, federal funds rate, government spending, nominal prices and also a proxy for financial flows.

3.3.1 Households

There is a continuum of homogeneous households indexed by $j \in [0,1]$, supplying specialized services $n_{j,t}$. They maximize the expected lifetime utility given by:

$$E_t \sum_{s=0}^{\infty} \beta^s \gamma_{t+s} \left[\frac{(c_{t+s} - hc_{t+s-1})^{1-\sigma}}{1-\sigma} - \alpha \frac{n_{j,t+s}^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}} \right] \tag{3.1}$$

where c is consumption, $n_{j,t}$ is labor of type j, and β is the discount factor. The variable γ_{t+s} evolves stochastically and captures shocks to the intertemporal margin, i.e., is a discount factor shock affecting both the marginal utility of consumption and the marginal disutility of labor. However, since it is an intertemporal shock it implies that it doesn't impact how households value utility from consumption today versus utility from leisure today, but rather how they value utility today versus utility in the future. For example, if γ_{t+s} increases, then households place relatively more weight on present utility than the future, i.e. they become more impatient. The parameter ε is the elasticity of labor supply i.e. the Frisch elasticity, and h determines the degree of "external" habit in consumption. The period utility is represented by $U(c_{t-1}, c_t, n_{j,t})$.

The household's budget constraint is:

$$w_{j,t}n_{j,t} + d_t + B_t + a_{j,t} = P_t q_t s_{t+1} + \frac{B_{t+1}}{1+r_t} + P_t c_t + T_t + \int q^{\omega}_{j,t+1} a_{j,t+1} d\omega_{j,t+1,1} \quad (3.2)$$

where r_t is the nominal rate on bonds and $w_{j,t}n_{j,t}$ is the nominal wage set by household j. The variable B_t is the one-period nominal bond, d_t is the equity payout received from the ownership of firms and T_t denotes nominal lump-sum taxes. Households can buy state-contingent claims $a_{j,t}$ at the price $q^{\omega}_{j,t+1}$ to insure against wage shocks.

Individual households are monopolistic suppliers of specialized labor and set the wage taking the demand function as given. The demand for labor of type j derives from the aggregation of the inputs demanded by all firms. The demand for type j labor, derived from the firm's problem described ahead, is given by:

$$n_{j,t} = \left(\frac{w_{j,t}}{W_t}\right)^{-\frac{\upsilon_t}{\upsilon_t - 1}} N_t \quad (3.3)$$

where N_t is the aggregate demand of labor and $W_t = \left(\int_0^1 w_{j,t}^{\frac{1}{1-\upsilon_t}} dj\right)^{1-\upsilon_t}$ is the aggregate nominal wage index. The variable υ_t is stochastic and captures shocks to the wage mark-up.

Households post nominal wages and supply the specialized services as determined by the demand function (3.3). Wage rigidities derive from the assumption that households can change their posted wage only with probability $1-\omega$ (Calvo's price rigidity).

Consider a household who is allowed to post a new wage in period t. Using the labor demand function (3.3), the new posted wage solves the following condition:

$$\max_{w_{j,t}} E_t \sum_{s=0}^{\infty} (\beta\omega)^s \gamma_{t+s} U\left(c_{t+s-1}, c_{t+s}, \left(\frac{w_{j,t}}{W_{t+s}}\right)^{-\dfrac{\upsilon_t}{\upsilon_t - 1}} L_{t+s} \right), \tag{3.4}$$

subject to the sequence of budget constraints (3.2).

After choosing $w_{j,t}$ at time t, the wage remains constant until the household is allowed to post a new wage. Only the periods preceding the resetting of a new wage are relevant for the choice of the wage today, explaining the probability ω in the discount factor.

Following the literature, we derive a wage equation by differentiating (3.4) with respect to $w_{j,t}$ and taking a log-linear approximation around the steady state. After some algebra, get the following log-linearized wage equation:

$$\hat{w}_t = -\left(\frac{h\sigma\Phi}{1-h}\right)\hat{c}_{t-1} + \left(\frac{\sigma\Phi}{1-h}\right)\hat{c}_t + \Phi\hat{P}_t + \Phi\hat{\upsilon}_t + \frac{\Phi}{\varepsilon}\hat{n}_t + \frac{\upsilon\Phi}{(\upsilon-1)\varepsilon}\hat{W}_t + \beta\omega E_t \hat{w}_{t+1} \tag{3.5}$$

where $\Phi = [\varepsilon(\upsilon-1)(1-\beta\omega)]/[\varepsilon(\upsilon-1)+\upsilon]$ and the hat sign denotes log-deviations from steady state.

Since all households that re-optimize choose the same $w_{j,t}$, the aggregate wage index evolves according to

$$W_t = \left[\omega W_{t-1}^{\frac{1}{1-\upsilon_t}} + (1-\omega) w_t^{\frac{1}{1-\upsilon_t}}\right]^{1-\upsilon_t} \tag{3.6}$$

In addition to the nominal wage and, implicitly, the supply of labor, households choose nominal bonds. The first order condition for B_{t+1} is:

$$1 = \beta(1+r_t) E_t \left(\frac{\gamma_{t+1} U_{2,t+1}}{\gamma_t U_{2,t}}\right)\left(\frac{P_t}{P_{t+1}}\right). \tag{3.7}$$

Firms' optimization is consistent with households' optimization and the stochastic discount factor is $m_{t+1} = \beta \left(\frac{\gamma_{t+1} U_{2,t+1}}{\gamma_t U_{2,t}} \right)$.

3.3.2 Firms

There is a continuum of firms in the $[0, 1]$ interval, each producing an intermediate good x_i. The intermediate good is used as an input in the final goods production, which is given by:

$$y_t = \left(\int_0^1 x_{i,t}^{\frac{1}{\eta_t}} di \right)^{\eta_t}. \tag{3.8}$$

The variable η_t is stochastic, capturing shocks to the nominal price mark-up.

The first order condition for the maximization of profits, $P_t y_t - \int_0^1 p_{i,t} x_{i,t} di$, returns the inverse demand function for the intermediate good i:

$$p_{i,t} = P_t y_t^{\frac{\eta_t - 1}{\eta_t}} x_{i,t}^{\frac{1-\eta_t}{\eta_t}}, \tag{3.9}$$

where $p_{i,t}$ is the nominal price set by the producer of good i and $P_t = \left(\int_0^1 p_{i,t}^{\frac{1}{1-\eta_t}} di \right)^{1-\eta_t}$ is the aggregate nominal price index.

The intermediate good is produced with capital and labor according to:

$$x_{i,t} = z_t (u_{i,t} k_{i,t})^\theta n_{i,t}^{1-\theta}, \tag{3.10}$$

where z_t is the aggregate productivity, $k_{i,t}$ the input of capital, $u_{i,t}$ the capital utilization rate and $n_{i,t} = \left(\int_0^1 n_{j,i,t}^{\frac{1}{\upsilon_t}} di \right)^{\upsilon_t}$ is the aggregation of all labor inputs used by firm i. The variable υ_t is stochastic and affects the demand elasticity for the different types of labor.

From the cost minimization problem we can derive the demand for labor of type j for each firm. Aggregating over all firms gives the aggregate demand for type j labor as reported in equation (3.3). Substituting the production into the inverse demand for the intermediate input, the price charged by firm i can be expressed as:

$$p_{i,t} = P_t Y_t^{\frac{\eta_t - 1}{\eta_t}} \left[z_t(u_{i,t}k_{i,t})^\theta n_{i,t}^{1-\theta}\right]^{\frac{1-\eta_t}{\eta_t}} \equiv P_t D(k_{i,t}, u_{i,t}, n_{i,t}; \mathbf{s}_t) \tag{3.11}$$

To take into account the dependence on the aggregate production Y_t, we have included the term $\mathbf{s}_t$, which is the vector of aggregate states.

Using (3.11) the real revenues of the firm can also be expressed as a function of the production inputs and aggregate states, that is:

$$p_{i,t}x_{i,t} = P_t Y_t^{\frac{\eta_t - 1}{\eta_t}} \left[z_t(u_{i,t}k_{i,t})^\theta n_{i,t}^{1-\theta}\right]^{\frac{1}{\eta_t}} \equiv P_t F(k_{i,t}, u_{i,t}, n_{i,t}; \mathbf{s}_t) \tag{3.12}$$

Physical capital is accumulated by firms and evolves according to $k_{t+1} = (1 - \delta)k_t + \Upsilon(i_{t-1}, i_t; \zeta_t)$, where ζ_t is a stochastic variable affecting the transformation of final goods in new capital goods (investment specific technology shock). The function $\Upsilon(i_{t-1}, i_t; \zeta_t)$ takes the form:

$$\Upsilon(i_{t-1}, i_t; \zeta_t) = \zeta_t \left[1 - g\left(\frac{i_t}{i_{t-1}}\right)\right] i_t, \tag{3.13}$$

with $g(1) = 0$, $g'(1) = 0$, $g''(.) > 0$. This cost function is not standard in the investment literature but has become popular in New Keynesian models. The function $g(i_t/i_{t-1})$ is specified as $\varrho(i_t/i_{t-1} - 1)^2$.

Capital utilization is also costly. Denoting by u_t the fraction of used capital over the owned capital, the utilization cost is $\Psi(u_t)k_t$ where we impose that $\Psi(1) = 0$, $\Psi'(1) > 0$ and $\Psi''(1) > 0$. The functional form for $\Psi(u_t)$ is specified as $\vartheta(u_t^{1+\psi} - 1)/(1 + \psi)$ where $\vartheta = (1 - \bar{\xi}\bar{\mu})/\beta - (1 - \delta)$ so that the steady state utilization is 1.

There are different ways of generating nominal price rigidity. A popular approach is based on Calvo's staggered prices which generates heterogeneity in firms' prices. The price heterogeneity can be easily handled in the case of complete markets. With incomplete markets, however, the characterization of the equilibrium is much more complex because the price heterogeneity generates heterogeneity in the financial structure of firms. Thus, we would not be able to aggregate and work with a 'representative firm'.

This problem does not arise with the Rotemberg's approach which is based on a convex cost of adjusting the nominal price. This is the only change we make to the model estimated by Smets and Wouters (2007), besides adding financial frictions and financial shocks.

Given the nominal price $p_{i,t-1}$ set in the previous period, the adjustment cost is:

$$G(p_{i,t-1}, p_{i,t}; \mathbf{s}_t) \equiv \frac{\phi}{2}\left(\frac{p_{i,t}}{p_{i,t-1}} - 1\right)^2 Y_t \tag{3.14}$$

We should think of the model as already detrended by long-term inflation.

The financial structure and frictions are the same as those described in the simpler model studied earlier. In particular, they are characterized by two parameters: τ and κ. The first parameter determines the tax advantage of using debt. Given r_t the nominal interest rate, the effective gross rate paid by firms is $R_t = 1+r_t(1-\tau)$. The second parameter determines the cost of changing the equity payout. Given the equity payout d_t received by shareholders, the cost for the firm is $\varphi(d_t) = d_t+\kappa(d_t-\bar{d})^2$. As in the simpler model, if we set these two parameters to zero, the model collapses to a New Keynesian model with complete markets.

The individual state variables for the firm are the nominal price chosen in the previous period, p_{-1}, the previous period investment, i_{-1}, the stock of capital, k, and the debt, b. Since in equilibrium all firms make the same choices (assuming that they start with the same states), from now on we omit the subscript i. The optimization problem is:

$$V(\mathbf{s}; p_{-1}, i_{-1}, k, b) = \max_{d,n,u,p,i,k',b'} \left\{d + Em'V(\mathbf{s}'; p, i, k', b')\right\} \tag{3.15}$$

subject to:

$$P\left[F(k, u, n; \mathbf{s}) - \Psi(u)k\right] + \frac{b'}{R} - b = Wn + PG(p_{-1}, p; \mathbf{s}) + P\varphi(d) + Pi$$

$$\xi\left(k' - \frac{b'}{P(1+r)}\right) \geq F(k,u,n;\mathbf{s})$$

$$\frac{p}{P} = D(k,u,n;\mathbf{s})$$

$$(1-\delta)k + \Upsilon(i_{-1}, i; \zeta) = k'$$

The problem is subject to the budget constraint, the enforcement constraint [8], the demand for the firm's product and the law of motion for capital. The first order conditions that solve the optimization problem faced by the firm represented in equation (3.15) are the following:

$$\lambda_t = \frac{1}{P_t \varphi_d(d_t)} \tag{3.16}$$

$$\left(1 - \frac{\mu_t}{\lambda_t P_t}\right) F_{l,t} = \frac{W_t}{P_t} + \frac{\chi_t D_{l,t}}{\lambda_t P_t} \tag{3.17}$$

$$\left(1 - \frac{\mu_t}{\lambda_t P_t}\right) F_{u,t} = \frac{\Psi_{u,t} k_t}{\lambda_t P_t} + \frac{\chi_t D_{u,t}}{\lambda_t P_t} \tag{3.18}$$

$$\lambda_t P_t G_{p,t} + E m_{t+1} \lambda_{t+1} P_{t+1} G_{p_{-1},t+1} = \frac{\chi_t}{P_t} \tag{3.19}$$

$$Q_t \Upsilon_{i,t} + E m_{t+1} Q_{t+1} \Upsilon_{-i,t+1} = \lambda_t P_t \tag{3.20}$$

$$Q_t = E m_{t+1} \left\{ (1-\delta) Q_{t+1} + \lambda_{t+1} P_{t+1} \Big(F_{k,t+1} - \Psi(u_t) \Big) - \mu_{t+1} F_{k,t+1} - \chi_{t+1} D_{k,t+1} \right\} + \xi_t \mu_t \tag{3.21}$$

[8] The enforcement constraint in this structural New Keynesian version of the Jermann and Quadrini (2012) model is derived in the same way than its Real Business Cycle counterpart, and it is originated from the renegotiation process between the firm and the lender to settle the amount of collateral in the event of firm's default on the intertemporal debt. A more detailed description of this process is included in the Appendix.

$$1 = R_t E m_{t+1} \frac{\lambda_{t+1}}{\lambda_t} + \frac{\mu_t \xi_t}{\lambda_t P_t} \left(\frac{R_t}{1 + r_t} \right) \tag{3.22}$$

Where λ, μ, χ, Q represent the Lagrange multipliers associated with the four constraints.

3.3.3 Public Sector

The government faces the following budget constraint:

$$P_t G_t + B_{t+1} \left(\frac{1}{R_t} - \frac{1}{1 + r_t} \right) = T_t \tag{3.23}$$

where G_t is real (unproductive) government purchases, r_t is the nominal interest rate and $R_t = 1 + r_t(1 - \tau)$ is the effective gross interest rate paid by firms. The cost of the interest deduction is $B_{t+1}/[1/R_t - 1/(1 + r_t)]$. Total expenditures are financed with lump-sum taxes T_t paid by households. Government purchases follow the stochastic process given by:

$$\widehat{G}_t = \rho_g \widehat{G}_{t-1} + \rho_{gz}(\hat{z}_t - \hat{z}_{t-1}) + \epsilon_{g,t} \tag{3.24}$$

where $\epsilon_{g,t} \sim N(0, \sigma_G)$.

Monetary policy takes the form of the following interest rate rule:

$$\frac{1 + r_t}{1 + \bar{r}} = \left(\frac{1 + r_{t-1}}{1 + \bar{r}} \right)^{\rho_R} \left[\left(\frac{\pi_t}{\bar{\pi}} \right)^{\nu_1} \left(\frac{Y_t}{Y_{t-1}} \right)^{\nu_2} \right]^{1-\rho_R} \varsigma_t \tag{3.25}$$

where ρ_R is a parameter that measures the degree of interest smoothing, ν_1 is the inflation coefficient, ν_2 is the output coefficient and $\varsigma_t \sim N(0, \sigma_R)$. The monetary authority targets inflation and output growth deviations from the steady state, and if we take this objective this into account we can rewrite the interest rate rule as follows:

$$\frac{1+r_t}{1+\bar{r}} = \left(\frac{1+r_{t-1}}{1+\bar{r}}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\nu_1} \left(\frac{Y_t}{Y^*}\right)^{\nu_2}\right]^{1-\rho_R} \left(\frac{\frac{Y_t}{Y_t^*}}{\frac{Y_{t-1}}{Y_{t-1}^*}}\right)^{\nu_3} \varsigma_t. \tag{3.26}$$

After log linearizing this equation in order to simplify it and isolate the nominal interest rate, we get the following condition:

$$a_1 r_{t-1} + a_2(P_t - P_{t-1}) + a_3(Y_t - Y_t^*) + a_4(Y_{t-1} - Y_{t-1}^*) + \varsigma_t - r_t = 0 \tag{3.27}$$

where $a_1 = \rho_R$, $a_2 = (1-\rho_R)\nu_1$, $a3 = (1-\rho_R)\nu_2 + \nu_3$, $a_4 = -\nu_3$.

Therefore, the zero lower bound constraint is implemented as a Taylor rule truncated at zero:

$$r_t = \max(Z_t, 0) \tag{3.28}$$

where

$$Z_t = a_1 Z_{t-1} + a_2(P_t - P_{t-1}) + a_3(Y_t - Y_t^*) + a_4(Y_{t-1} - Y_{t-1}^*) + \varsigma_t \tag{3.29}$$

3.3.4 Stochastic Processes for the Shocks

The stochastic processes regarding the fiscal policy (represented through a shock over government purchases) and the monetary policy (captured through a shock over the nominal interest rate) have already been specified in conditions (3.24) and (3.25). The remaining stochastic variables follow the generic process $\hat{x}_{t+1} = \rho_x \hat{x}_t + \epsilon_{x,t+1}$, with $\epsilon_x \sim N(0, \sigma_x)$ and $x \in \{z, \zeta, \gamma, \eta, \upsilon, \xi\}$. The hat sign denotes log deviations from steady state.

3.3.5 Equilibrium

An equilibrium in this economy is a sequence of stochastic processes:

$$\{c_t, n_t, u_t, \mu_t, \chi_t, x_t, Q_t, R_t, P_t, Q_t, i_t, w_t, W_t, Y_t, V_t, T_t, k_{t+1}, b_{t+1}\}$$

such that for any given sequence of states:

$$\{z_t, \zeta_t, \gamma_t, \eta_t, \upsilon_t, G_t, k_t, b_t, \varsigma_t, \xi_t, p_{t-1}, i_{t-1}, c_{t-1}, W_{t-1}, R_{t-1}, Y_{t-1}\}$$

the household and firm problems are satisfied, the monetary and fiscal policy rules are satisfied, and markets clear. Conditions (3.2), (3.5), (3.6) and (3.7) characterize the household problem, while conditions (3.16)-(3.22), combined with conditions (3.12), (3.13), (3.14) and the maximization problem (3.15) characterize the equilibrium through the firm's problem. That collection of state variables also satisfies the government budget constraint (3.23) and the monetary policy rule given by equation (3.26). In equilibrium the debt repurchases must satisfy the following condition:

$$x_t = \frac{1}{Y_t}\left(\frac{b_t}{1+r_{t-1}} - \frac{b_{t-1}}{1+r_t}\right) \tag{3.30}$$

The full set of equations that characterize this equilibrium is presented in section 3.5.1.1 of the Appendix.

3.3.6 Estimation

A small number of the model parameters are set using the standard calibration technique based on steady state targets described in the simpler real business cycle model proposed by Jermann and Quadrini (2012), which includes only two shocks. The remaining parameters are estimated using Bayesian methods as described in An and Schorfheide (2007).

However, the calibration of some of the parameters incorporates the correction proposed by Pfeifer (2016) in a paper where the author shows that the Jermann and Quadrini (2012) model suffers from methodological issues which affect the accuracy and the consistency of the estimates. According to Pfeifer (2016), "the constructed

TFP combines GDP of the total economy with inputs pertaining to the business sector only", and the author also shows that the structural approach is affected by issues in three equilibrium conditions. Additionally, Pfeifer (2016) remarks that Jermann and Quadrini (2012) "make a timing error by using the end of period rather than the beginning of period capital stock as an input, which is inconsistent with their model". Correcting these issues changes the calibration of several parameters, leading to crucial alterations in the estimation of the shocks series (e.g. a more volatile TFP series) and in the main findings of the model. Pfeifer (2016) concludes that, after recalibrating the model to match the corrected calibration targets, the technological shock accounts for the 2008 financial crisis, while the contribution of the financial shock is considerably muted. Although its role during the recent Great Recession is more pronounced than without the corrections, it still accounts for approximately a third of the percentage GDP decrease occurred in this period.

3.3.6.1 Calibrated Parameters

This is a quarterly model and the calibration targets for the few calibrated parameters are the same as those presented in the real business cycle version of the Jermann and Quadrini (2012) model.

More specifically, those parameters are set as: $\beta = 0.9825$, $\tau = 0.35$, $\delta = 0.025$ and α is chosen to have an average working time of 0.3. The average value of the enforcement variable $\bar{\xi}$ is chosen to have a steady state ratio of debt over quarterly output of 3.36. Finally, the average value of government purchases $\bar{G}$ is chosen to have a steady state ratio of government purchases over output of 0.18.

3.3.6.2 Estimated Parameters

As in Jermann and Quadrini (2012), the model is estimated using eight empirical series: growth rate of GDP, growth rate of personal consumption expenditures, growth rate of private domestic investment, growth rate of implicit price deflator for GDP, growth rate of working hours in the private sector, growth rate of hourly wages in the business sector, federal fund rate and debt repurchases in the nonfinancial business sector. The first seven variables are similar to the variables used in Smets and Wouters (2007), but debt repurchases are added because there is an additional shock in the model, ξ. The sample period is 1984.I-2010.II. We start in 1984 to avoid the issue of possible structural breaks associated with the so-called 'great moderation.' See the appendix for a more detailed description of the data.

To generate artificial series, the model is solved numerically after log-linearizing around the steady state. This is possible because the enforcement constraint is always binding in the neighbourhood of the steady state equilibrium. The whole set of equations are listed in the appendix.

The choice of the prior distributions are the same as those used in Smets and Wouters (2007) with the exception, of course, of the parameters that were not present in that model. In particular, the parameters that govern the stochastic process for the financial shock, ρ_ξ and σ_ξ, and the flexibility in equity payout, κ. For the persistence and standard deviation of the financial shocks we use the same priors as those used for the other shocks. For the parameter κ we use an inverse gamma distribution with a mean of 0.146 (the calibration value used in the simpler model) and a standard deviation of 0.05.

Table 3.1 reports the parameters, calibrated according to the Jermann and Quadrini (2012) methodology and the estimated parameters computed with Bayesian techniques, following the Jermann and Quadrini (2012) structural approach based in the Smets and Wouters (2007) model. However, the table displays the posterior estimation results of Pfeifer (2016) (the mode and the cutoff values for the 5 and 95 percentiles of the posterior distribution), and only the posterior mode is compared against the Jermann and Quadrini (2012) results [9].

[9] Pfeifer (2016) and Jermann and Quadrini (2012) share the same prior densities.

Table 3.1: Parameterization

Calibrated Parameters	Par	Value
Discount factor	β	0.9825
Tax advantage	τ	0.3500
Utility parameter	α	16.7360
Production technology	θ	0.3600
Depreciation rate	δ	0.0250
Enforcement parameter	$\bar{\xi}$	0.1990
Average government purchases	$\bar{G}$	0.179

Estimated Parameters		Prior Distribution			Posterior Distribution			
					JQ	Reestimation		
	Par.	Dist.	Mean	S.D.	Mode	Mode	5%	95%
Risk aversion	σ	Normal	1.500	0.370	1.090	1.540	0.855	1.731
Frisch elasticity (of labor)	ε	Normal	2.000	0.750	1.761	0.873	0.940	2.998
Habit in consumption	h	Beta	0.500	0.300	0.608	0.367	0.263	0.500
Calvo wage adjustment	ω	Beta	0.500	0.300	0.278	0.075	0.037	0.220
Rotemberg price adjustment cost	ϕ	InvGamma	0.100	0.300	0.031	6.997	7.300	29.584
Investment adjustment cost	ϱ	InvGamma	0.100	0.300	0.021	0.149	0.102	1.371
Capital utilization cost	ψ	Beta	0.500	0.150	0.815	0.775	0.548	0.882
Equity payout cost	κ	InvGamma	0.200	0.100	0.426	0.287	0.254	0.935
Average price markup	$\bar{\eta}$	Beta	1.200	0.100	1.137	1.806	1.712	1.871
Average wage markup	$\bar{\upsilon}$	Beta	1.200	0.100	1.025	1.140	1.057	1.374
Productivity shock persistence	ρ_z	Beta	0.500	0.200	0.902	0.920	0.864	0.949
Investment shock persistence	ρ_ζ	Beta	0.500	0.200	0.922	0.913	0.623	0.928
Intertemporal shock persistence	ρ_γ	Beta	0.500	0.200	0.794	0.949	0.920	0.979
Price markup shock persistence	ρ_η	Beta	0.500	0.200	0.906	0.866	0.734	0.910
Wage markup shock persistence	ρ_υ	Beta	0.500	0.200	0.627	0.981	0.945	0.996
Government shock persistence	ρ_G	Beta	0.500	0.200	0.955	0.976	0.957	0.993
Interest policy shock persistence	ρ_ς	Beta	0.500	0.200	0.203	0.213	0.131	0.338
Financial shock persistence	ρ_ξ	Beta	0.500	0.200	0.969	0.990	0.978	0.998
Interaction production government	ρ_{gz}	Beta	0.500	0.200	0.509	0.859	0.608	0.969
Taylor rule persistence	ρ_R	Beta	0.750	0.100	0.756	0.784	0.767	0.849
Taylor rule feedback	ν_1	Normal	1.500	0.250	2.410	2.202	1.984	2.505
Taylor rule feedback	ν_2	Normal	0.120	0.050	0.000	-0.020	-0.032	0.050
Taylor rule feedback	ν_3	Normal	0.120	0.050	0.121	0.176	0.141	0.232
Technology shock	σ_z	InvGamma	0.001	0.050	0.005	0.005	0.004	0.005
Investment shock	σ_ζ	InvGamma	0.001	0.050	0.006	0.009	0.007	0.049
Preference shock	σ_γ	InvGamma	0.001	0.050	0.016	0.019	0.013	0.028
Price markup shock	σ_η	InvGamma	0.001	0.050	0.019	0.013	0.013	0.031
Wage markup shock	σ_υ	InvGamma	0.001	0.050	0.085	0.021	0.012	0.022
Government shock	σ_g	InvGamma	0.001	0.050	0.028	0.016	0.014	0.018
Monetary shock	σ_ς	InvGamma	0.001	0.050	0.002	0.001	0.001	0.002
Financial shock	σ_ξ	InvGamma	0.001	0.050	0.008	0.016	0.013	0.018

Source: Pfeifer (2016) and Jermann and Quadrini (2012).

However, since both calibration schemes were not able to generate shocks large enough to activate the zero lower bound in the simulated economies, several parameters were recalibrated to increase the friction of the shocks: the persistence of the preference shock, ρ_γ, was reset to 0.95; the Taylor rule persistence parameter, ρ_R, increased to 0.9; and the Taylor rule feedback parameter, ν_3, increased to 0.6. It is important to take into account that increase the value of ν_3 implies that the Taylor rule is increasing the weight on the ratio between the current output gap Y_t/Y_t^* relative to the output gap in the previous period, Y_{t-1}/Y_{t-1}^*, which helps to propagate and amplify the impact of each shock over the economy. When using the Smets and Wouters (2007) framework to analyze the post-crisis inflation and employment, Fratto and Uhlig (2014) concluded that, although the zero lower bound resulted in a tightening monetary policy from 2008 to 2010, the magnitude and duration of this effect was actually lower than most studies suggest. Fratto and Uhlig (2014) decomposed the nominal interest rate according to the Taylor rule from the model (which is identical to conditions (3.26) and (3.29)) and concluded that the term related to the "change-in-the-ouput-gap term is key, while the output-gap term itself is not". In other words, even if the output gap $\left(\frac{Y_t}{Y^*}\right)$ is large, its direct effect on the interest rate dynamics is null because, in this case, the parameter of the Taylor rule associated with it is set to zero ($\mu_2 = 0$). Therefore, the output gap growth rate, $\left(\frac{\frac{Y_t}{Y_t^*}}{\frac{Y_{t-1}}{Y_{t-1}^*}}\right)$, is the main responsible for the initial drop of the nominal interest rate below zero after a shock hits the economy and for its subsequent increase. It is also the main responsible to bind the lower zero bound when a shock hits the economy and increase its spell over time. Hence, this finding validates the need to increase the calibration of the parameter μ_3 associated with the output gap growth component to a higher value ($\mu_3 = 0.6$) than the original value proposed by Jermann and Quadrini (2012) ($\mu = 0.121$) or even Pfeifer (2016) ($\mu_3 = 0.176$).

After incorporating these changes, the simulations were conducted under the original Jermann and Quadrini (2012) and under the Pfeifer (2016) updated calibration scheme for the remaining parameters.

3.3.7 Simulation

In order to impose the zero lower bound constraint into the Jermann and Quadrini (2012) model, the model is simulated using Occbin, a library of numerical routines developed by Guerrieri and Iacoviello (2015) that solves the model through a piecewise linear solution algorithm compatible with Dynare. This model includes only one occasionally binding constraint, which implies that the nominal interest rate cannot be lower than zero (the ZLB constraint). In such cases, the Occbin algorithm is designed to assume the existence of two regimes: one where the occasionally binding constraint is slack and another one where the constraint is binding. The model is linearized under each regime around the non-stochastic steady state, and choose the regime [10] that applies at the point of linearization as the "reference" regime and the other regime as the "alternative." It is important to stress that the Occbin algorithm demands the fulfilment of two requirements to ensure its correct implementation: the reference regime must ensure that the conditions for existence of a rational expectations solution recommended by Blanchard and Kahn (1980) hold; and that "if shocks move the model away from the reference regime to the alternative regime, the model will return to the reference regime in finite time under the assumption that agents expect that no future shocks will occur".

When applying the Occbin toolkit developed by Guerrieri and Iacoviello (2015) to compare the results of the model with and without a binding zero lower bound constraint, both scenarios are simulated with a first order linear approximation around the steady state, instead of using the Bayesian methods used by Jermann and Quadrini (2012) and Pfeifer (2016), assuming that the enforcement constraint is always binding around the steady state for $\tau > 0$, similarly to the parsimonious RBC model with only two shocks initially developed by Jermann and Quadrini (2012).

In this simulation exercise the economy without a binding zero lower bound, i.e.,

[10] Guerrieri and Iacoviello (2015) emphasize that it is irrelevant whether the occasionally binding constraint is binding at the reference regime or the alternative regime.

the economy where the monetary policy follows a nontruncated Taylor rule so that the nominal interest rate is allowed to float freely, it is designated as the benchmark economy.

3.4 Findings

3.4.1 Variance Decomposition

Before analysing the impact of different shocks over the path of the main variables of this structural model, it is important to inspect the variance decomposition for the eight shocks used in the estimation (reported in Table 3.2), and establish the main differences between these findings and the results obtained by Jermann and Quadrini (2012) and Pfeifer (2016). In this case, the shock that contribute the most to the volatility of the growth rate of output is clearly the price markup shock (46.26%), followed by the technology shock (11.27%), and the preference shock (8.95%). The financial shock only accounts for 4.26% of the total volatility of output, contributing only marginally to the volatility of consumption (2.56%). The price markup shock is also the major driving force behind the volatility in investment (37.55%), hours (34.25%), interest rate (35.85%)and debt repurchases (26.07%). Besides nominal price markups shocks, preference shocks (21.64%) and the marginal efficiency of investment (MEI) shocks (16.56%) also play a large role in explaining the forecast error variance of consumption. Surprisingly, the contribution of financial shocks is rather small for the volatility of all variables, and its highest contribution concerns movements in debt repurchases, only reaching 7.18%. Generally, the contributions of intertemporal preference shocks is also small, although reaches a sizable size regarding the volatility in consumption (21.64%), investment (17.33%) and the interest rate (10.85%). These results differ substantially from the Jermann and Quadrini (2012) original estimation (see Table 3.6) in the Appendix) and the Pfeifer (2016) reestimation (see Table 3.7) in the Appendix), although they are more consistent with the latter. Indeed, in this case movements in output are largely explained by price markup innovations (as in the Pfeifer (2016) estimation) and much less by financial shocks. However, according to Pfeifer (2016) results the contribution of the MEI shock is also considerable (26.11%) to explain the GDP growth rate fluctuations. The price markup shock is the major driving force for the volatility of almost all variables, especially to explain debt repurchases movements (68.71%). Jermann and Quadrini (2012) rationalize this result through the link between the price markup shock and the enforcement constraint.

It is important to take these results into account regarding the variance decomposition of errors before analysing the impulse response functions and simulated series generated by the model, since the impact of some shocks over the economy is stronger than others, and that can substantially bind the response of the main variables of the model to a particular shock.

Table 3.2: Variance Decomposition (Jermann and Quadrini (2012) original calibration except for $\rho_\gamma = 0.95$, $\rho_R = 0.9$, $\nu_1 = 2.410$, $\nu_2 = 0$, $\nu_3 = 0.6$)

	TFP Shock z	**Investment Shock** ζ	**Preference Shock** γ	**Price MK Shock** η	**Wage MK Shock** υ	**Government Shock** G	**Money Shock** ς	**Financial Shock** ξ
GDP	11.27	3.17	8.95	46.26	7.87	5.2	7.13	4.26
Consumption	7.41	16.56	21.64	32	3.92	15.11	2.56	1
Investment	9.23	12.76	17.33	37.55	8.26	3.25	6.07	2.54
Hours	10.98	5.78	8.02	34.25	13.66	9.75	10.3	4.24
Wages	4.9	9.56	3.96	15.54	4.24	6.91	34.82	6.47
Interest Rate	9.23	7.71	10.85	35.85	11.42	3.08	9.85	6.69
Inflation	5.25	8.8	3.94	15.34	1.76	6.09	34.95	6.6
Debt Repurchases	5.65	12.93	7.12	26.07	2.99	4.45	19.29	7.18

Note: numbers do not add up to 100 due to non-zero correlation of simulated shocks in small samples.

3.4.2 Dynamic Analysis

3.4.2.1 Effect of a Preference Shock

At time zero, the economy is hit with a 2 standard deviation negative shock over the intertemporal preference parameter, γ_t, which lasts for 10 quarters, assuming that, at time zero, the economy was in its nonstochastic steady state [11]. The shock is large enough so that the zero lower bound on the nominal interest rate binds in periods five through ten. Figure 3.1 displays the dynamic path of the economy after the shock hits the economy and the interest rate subsidy is fixed in its steady state value ($\tau = 0.35$). It establishes a comparison between the impulse response functions for key variables of an economy constrained by the zero lower bound (i.e. the monetary policy follows a truncated Taylor rule $r_t = \max(Z_t, 0)$) and of an economy where the nominal interest rate is allowed to float freely (i.e. the monetary policy follows a nontruncated Taylor rule allowing for negative nominal interest rates).

The shock leads to a substantial decline in consumption, inflation nominal wages and in the nominal interest rate (which hits the zero lower bound in periods five through ten). Although output and hours also decrease slightly when the shock hits, when the ZLB binds in period five both variables start to exhibit an increasing behavior which lasts until period fifteen, when both variables start to converge once more to the steady state. Contemporaneously, only investment responds positively to this shock, since households consume less and increase disposable savings that can spur an investment boom.

[11]As Adolfsen (2017) points out, in the literature, modelling a liquidity trap through a large preference shock is a relatively standard way of studying the policies aimed at solving the ZLB problem (see for example Correia et al. (2013), Carlin and Soskice (2018), Chari et al. (1991), Christiano et al. (2011), Eggertsson (2009), etc) essentially due to its simplicity and tangibility when isolating the effects from the ZLB on fiscal multipliers. However, Adolfsen (2017) simulated a 7 standard deviations negative preference shock because by simulating with a single one standard deviation preference shock is "an unrealistic way of modelling a liquidity trap", and is unable to generate large ZLB spells.

In general, the impulse responses for all variables when the zero lower bound is binding are slightly larger relative to the paths achieved in the economy without a binding ZLB. To justify this behavior, Guerrieri and Iacoviello (2015), based on Nakata (2017) and other authors, emphasize that there are two main forces shaping these differences between economies with and without a binding zero lower bound: an uncertainty effect and a price dispersion effect. The uncertainty effect implies that "negative shocks at the ZLB produce larger contractions than away from it, since monetary policy is unable to offset them". On the other hand, if agents anticipate a negative shock when the economy is already at the ZLB then that shock will further reduce prices and output, since agents expect that monetary policy will be unable to overcome the consequences of that type of shock over the economy. Therefore, when that uncertainty is explicitly taken into account into the model, the ZLB binds more frequently, monetary and fiscal policies become more accommodative, and output produces larger impulse responses when hit by an unexpected shock. The uncertainty effect leads then to larger impulses responses at the zero lower bound than captured by the more standard economy allowing for negative nominal interest rates, which ignores that uncertainty. In turn, Guerrieri and Iacoviello (2015), based on Braun et al. (2012) and Braun et al. (2013), argue that the price dispersion effect implies that the unpredictability of the impulse responses generated by the economy without a binding ZLB are due to nonlinearities in the equilibrium conditions of the model that can be "important especially for large shocks that take output close to the ZLB". In this model, this dispersion is captured both through prices, whose nominal rigidity is generated with the Rotemberg's approach, based on a convex cost of adjusting nominal prices (equations (3.12) and (3.14)) and through wages, which are derived from the assumption that households can decide to change their posted nominal wage with probability $1 - \omega$ (Calvo's price rigidity) (equations (3.3) and (3.6)). Equation (3.6) (the nominal wage index), for example, can be interpreted as a measure of the evolution of wage dispersion. If the price or wage dispersion is high, then a fraction of the firms, stuck with lower prices, are inefficiently capturing a disproportionate fraction of aggregate demand. In Figure 3.1, wage dispersion leads nominal wages to decrease more in the benchmark case without ZLB, which can be explained by the temporary drop of the inefficiency caused by staggering wages, and cushions the decrease in wages in the economy with a binding zero lower bound in relation to the benchmark economy. This so called price dispersion effect partly offsets the dynamic induced by the uncertainty effect when a shock hits the economy. This result holds for a preference shock or any other type of shock.

Although all the impulse response functions in both economies (with and without a binding ZLB) coincide when the shock hits, the dynamic path followed by all variables in each economy start to diverge slightly after the zero lower bound binds in period five. However, that divergence is more pronounced for the nominal and real interest rate, wages and inflation. For output, hours and investment that divergence lasts for several periods but tends to disappear when these variables start to return to the steady state. Only consumption exhibits an almost coincident path with or without a binding ZLB. In general, when the zero lower bound binds, the responses of output, hours and investment in the constrained economy are steeper than the responses in the benchmark economy, although that gap is gradually reduced as each variable return to its steady state. The difference between the IRF of the benchmark economy and the IRF originated in the economy with a binding ZLB is particularly prominent in the case of inflation, wages and the real interest rate, and in opposition with the previous variables, that gap does not disappear as the simulation horizon increases, but tends to stabilize.

To explain this intertemporal behavior, it is important to highlight that a temporary decrease in γ_t is equivalent to a temporary increase in the discount factor β, since households now value current utility relatively less than future utility, i.e. they become more patient and they want less consumption and leisure today, therefore justifying the decrease in c_t and an increasing path for labor throughout the simulation horizon (although the contemporaneous response of labor is slightly negative). This dynamic can also be justified from the FOC for labor: the decrease in consumption shifts the labor supply curve out. An outward shift of the labor supply along a stable labor demand leads to a decrease in wages. Besides, the increasing path of hours worked, without an immediate adjustment in technology z_t or in the stock of capital k_t implies an upwards response of output along the simulation horizon after the initial sudden fall. Initially, after the shock hits the economy, output and hours drop together with consumption because the shock initially increases price markups compared to their steady state level. This leads to a drop in inflation in producer prices.

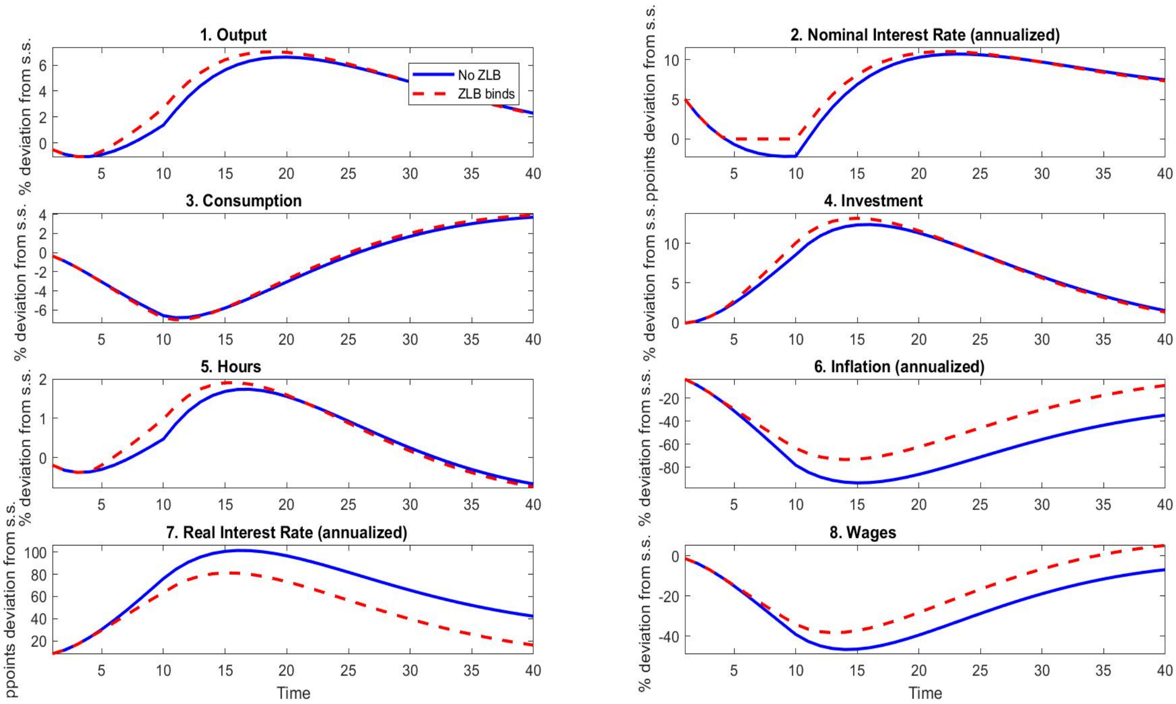


Figure 3.1: IRFs to a Preference Shock - $\tau = 0.35$

Notes: Time, measured in quarters, in represented on the horizontal axis. Output, consumption, investment, hours and wages are measured in log deviations from steady state (in percentage). The nominal interest rate, the real interest rate and inflation are measured in percentage points deviations from the steady state, in annualized levels.

3.4.2.1.1 The impact of the interest rate subsidy (τ) over the dynamic path of the variables

Figure 3.2 displays the impulse response functions of the nominal interest rate, output, consumption, hours, investment and wages after a negative preference shock that lasts ten periods hits the economy for different values of the interest rate subsidy, τ. The model is simulated for five distinct scenarios: in an economy where the Taylor rule is truncated so that $r_t = \max(Z_t, 0)$, i.e. the zero lower bound is binding, the model is simulated for four different values of the interest rate subsidy: $\tau = 0.0001$, $\tau = 0.2$, $\tau = 0.35$ (the steady-state value), and $\tau = 0.5$. These scenarios are compared against the benchmark economy, where the Taylor rule allows for negative nominal interest rates and therefore the zero lower bound is not binding, with the interest rate subsidy set at its steady state value: $\tau = 0.35$. The main purpose of this simulation exercise is to infer if the utilization of the interest rate subsidy as a fiscal policy instrument allows to dampen or even neutralize completely the frictions caused by the presence of a binding zero lower bound and replicate the effects of negative nominal interest rates or, on the contrary, if the presence of this subsidy enhances the effects of the zero lower bound constraint.

It is important to take into account that, in the Jermann and Quadrini (2012) framework, the presence or omission of the tax benefit τ affects directly the steady state values of almost all endogenous variables of the model. In fact, according to Jermann and Quadrini (2012), if the interest rate subsidy τ and the equity payout parameter κ are simultaneously set to zero, then the model collapses to a New Keynesian model with complete markets. This is essentially due to the fact that if $\tau > 0$ the enforcement constraint binds in a steady state, implying that a changing value of τ would lead to major alterations in the steady state levels of the variables of the simulated models.

In Figure 3.2 we can observe that for all cases considered the path followed by all variables after the preference shock hits the economy is very similar, but the magnitude of the responses changes considerably along with the value of τ. In the economy where the monetary policy allows for negative nominal interest rates, with an interest rate tax benefit of 35% (the benchmark economy), the magnitude of the impulse responses of all endogenous variables are greater than the responses of the allocation that arises in the economies where the zero lower bound constraint is binding, except for the case where the interest rate subsidy is large enough ($\tau = 0.5$). In the benchmark economy, the preference shock produces wider impulse responses for output, consumption, investment, nominal interest rate, and, on a smaller scale, for hours and wages. For output, the path is smoother for smaller values of τ throughout all the simulation horizon: in the first five quarters, after the shock hits the economy, the negative impact is stronger for higher values of τ in the economies where the ZLB binds and in the benchmark case. After that initial period, when the path inverts, the increase in output is much smoother for lower values of τ, particularly when $\tau = 0.0001$. The paths converge to the same steady state for almost all cases, except for $\tau = 0.5$, that seems to decline at a much higher rate than its counterparts, and keeps that gap throughout the whole simulation horizon. Investment and hours worked follow a very similar path to the impulse response followed by output, although in the investment case there is an immediate positive response to the preference shock. This result, although counter intuitive with the standard decline in investment that economic theory predicts in the presence of a negative preference shock, comes from the fact that with a negative preference shock households are more patient and desire to consume less goods and services and less leisure (therefore increasing hours worked), and that drop in consumption increases disposable savings that can lead to an investment boost, although the real interest rate increases both in presence of a binding ZLB or otherwise.

In general, the higher the interest rate tax benefit τ is, the wider is the impulse response of each variable. Therefore, a higher tax benefit τ associated with the presence of a binding zero lower bound constraint contributes to amplify the impact of the preference shock over the economy and enhances the impulse responses of the majority of the endogenous variables. This result also holds in the case of an economy without a binding ZLB. In order to smooth the effects of the preference shock, the fiscal policy authorities can reduce the interest rate tax benefit, and if it is small enough and very close to zero ($\tau = 0.0001$) [12], the zero lower bound constraint does no longer binds, but the magnitude of the impulse response functions are much smaller than in the benchmark case, implying that using the interest rate subsidy

[12]When $\tau = 0$, the economy does not necessarily collapses to a New Keynesian benchmark model with complete markets, only when $\tau = 0$ and $\kappa = 0$, but the absence of the interest rate subsidy is crucial to the simulations of the model since it changes the steady state values and consequently, the dynamics of the model.

as the only fiscal instrument to circumvent the zero lower bond effects is not enough to replicate the first best allocation of the benchmark economy.

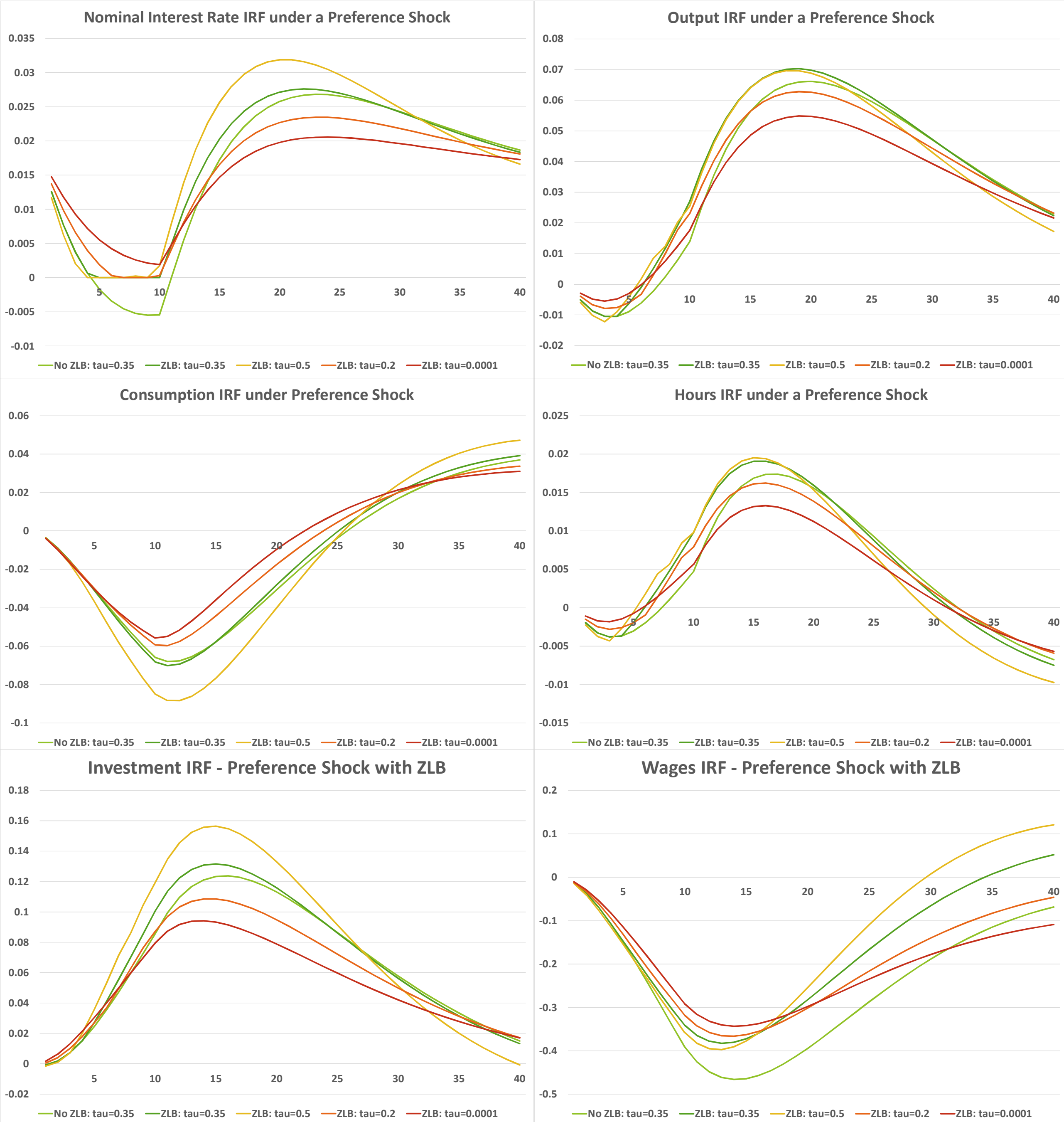


Figure 3.2: IRFs to a Preference Shock

Notes: Time, measured in quarters, in represented on the horizontal axis. Output, consumption, investment, hours and wages are measured in log deviations from steady state (in percentage). The nominal interest rate, the real interest rate and inflation are measured in percentage points deviations from the steady state, in annualized levels.

3.4.2.2 Effect of a Financial Shock

Although according to Table 3.2 the contribution of financial shocks to the volatility of output is quite small (4.26%), that corroborates the findings of Pfeifer (2016) (6.53%) and contradicts the large contribute of 46.4% reported in Jermann and Quadrini (2012). Still, the impact of financial shocks over the business cycle movements of the economy cannot be understated [13], neither in size nor in the dynamics of the variables. Lindé et al. (2016) concluded that adding financial extensions or financial shocks to a structural model such as Smets and Wouters (2007) does not add much propagation of other macroeconomic shocks, and therefore those frictions are not sufficient to properly analyze or explain the effects of non-standard monetary policy (e.g. quantitative easing measures) and macroprudential policies. However, they argue that those extensions are able to account for features of the 2008 financial crisis and subsequent recession if the models appropriately can "integrate the non-linear accelerator dynamics from financial frictions". Other studies, such as Negro et al. (2017), which also incorporate financial disturbances into a standard DSGE New Keynesian model, concluded that a liquidity shock [14], calibrated to match the increase in the premium associated with very liquid assets during the 2008 financial crisis accounts for more than half of the drop in output observed in the data and all of the drop in inflation. These findings show that accounting for shocks originated in the financial sector is especially relevant in the post-crisis period, since those studies have proved that the effects of those shocks over the real economy can be quite large.

However, it must be stressed that the financial shock and the financial frictions incorporated in the Jermann and Quadrini (2012) setup, although designed to affect directly the financial sector of the economy, have a different nature and do not explicitly address the financial intermediation sector as in Lindé et al. (2016) (where,

[13] Pfeifer (2016) estimated that despite the small contribution of financial shocks to explain output volatility, these shocks are responsible to 2-3 percentage points of the observed GDP fall during the Great Recession.

[14] Defined as a constraint imposed on firms to sell only up to a certain fraction of the illiquid assets on its balance sheet in each period.

based on the basic approach of Bernanke et al. (1999), the intermediate goods producers rent capital services from entrepreneurs rather than directly from households) or as in the approach followed by Negro et al. (2017), in which the financial and the liquidity shocks have its origins in the money market and in the secondary market.

3.4.2.2.1 The impact of the interest rate subsidy (τ) over the dynamic path of the variables

Figure 3.3 plots the impulse response functions of output, consumption, hours, investment, the nominal interest rate and wages after a 4 standard deviations negative financial shock, simulated for the economy without a truncated Taylor rule (no zero lower bound) for $\tau = 0.35$, and for the economies with the binding ZLB for four [15] different values of the interest rate tax subsidy ($\tau = 0.0001$, $\tau = 0.2$, and $\tau = 0.35$).

For all the cases considered, output, hours, investment, and wages fall immediately after the financial shock hits, although those drops are smaller for lower values of the tax benefit, τ, which are in line with the Jermann and Quadrini (2012) and the Pfeifer (2016) responses. Jermann and Quadrini (2012) argue that the large drop in hours worked is the result of the strict link between the tightness of the enforcement constraint and the condition (3.38) that defines labor demand [16]. In equation (3.38), μ is the multiplier for the enforcement constraint and the term $\mu\varphi_d(d)$ defines the labor wedge, i.e. a wedge between the wage and the marginal product of labor in the demand for labor. When a financial shock hits the economy the enforcement

[15] For this particular shock, the simulation for the economy with a binding ZLB and $\tau = 0.5$ will be omitted in this section because of this high instability and strong volatility of the responses of all variables in that case. Since the magnitude of those responses is very large comparatively to the size of the responses obtained for the other simulations, and the inclusion of the simulation with a binding ZLB and $\tau = 0.5$ hinders the comparison between the other simulations, that specific simulation is omitted from Figure 3.3, but can be consulted in Figure 3.19 in the appendix.

[16] Although Jermann and Quadrini (2012) explained this mechanism in the context of the Real Business Cycle (RBC) setting of their model, the argument it is also valid in the context of the structural approach of their model.

constraint becomes tighter (higher multiplier μ), which increases the labor wedge and will function much like an increase in a time-varying tax on labor income. This implies that, in order to keep the same scale and maintain the same level of hours of work, firms have to reduce the equity payout distributed to shareholders, and since this is costly, firms partially choose to reduce the equity payout and partially the input of labor.

The nominal interest rate drops to negative values in the benchmark economy, and in the remaining simulations the duration of the ZLB increases with the tax subsidy, τ. In relation to the economies with an active ZLB, the negative responses of almost all variables in the benchmark economy are much deeper, especially for investment and wages. This is consistent with the fact that a higher interest rate subsidy implies a tighter enforcement constraint (a higher multiplier μ), which in turn increases the labor wedge and the distortionary effect similar to a tax on labor income. In other words, a higher tax benefit, combined with the presence of price and wage rigidities and investment adjustment costs, seems to have the effect of increasing propagation and persistence of the financial shock.

The only exception to this almost unanimous fall after the shock is consumption, whose impulse responses in almost all simulations are more positive the higher is the tax benefit, τ. The economy with a binding ZLB and an almost absent interest rate subsidy ($\tau = 0.0001$) corresponds to the only case where consumption decreases after the financial shock hits. This behavior of consumption is related to the result that, in the structural setup of the Jermann and Quadrini (2012) model, when the parameters τ (which defines the interest rate subsidy) and κ (which defines the equity payout cost) are both set to zero, then the model collapses to a standard New Keynesian model with complete markets. Rupert and Sustek (2019) argue that in a model with endogenous capital and sticky prices and wages, when output temporarily drops as a response to a contractionary shock, smooth consumption can be achieved by reducing investment and adjusting the capital stock. These authors also argue that in economies that exhibit a high persistence of the Taylor rule (set to $\rho_R = 0.9$ in this case), consumption initially rises higher the larger ρ_R is, before declining below the steady state after the shock fades away, as can be observed in Figure 3.3. In this also important to note, by inspecting Figure 3.4, that inflation increases sightly after the financial shock hits, especially in the economy with a binding ZLB. This leads to a small drop in the real interest rate in both cases, more pronounced in the economy where the ZLB binds, which in turn makes the positive response of consumption lower in the constrained simulation.

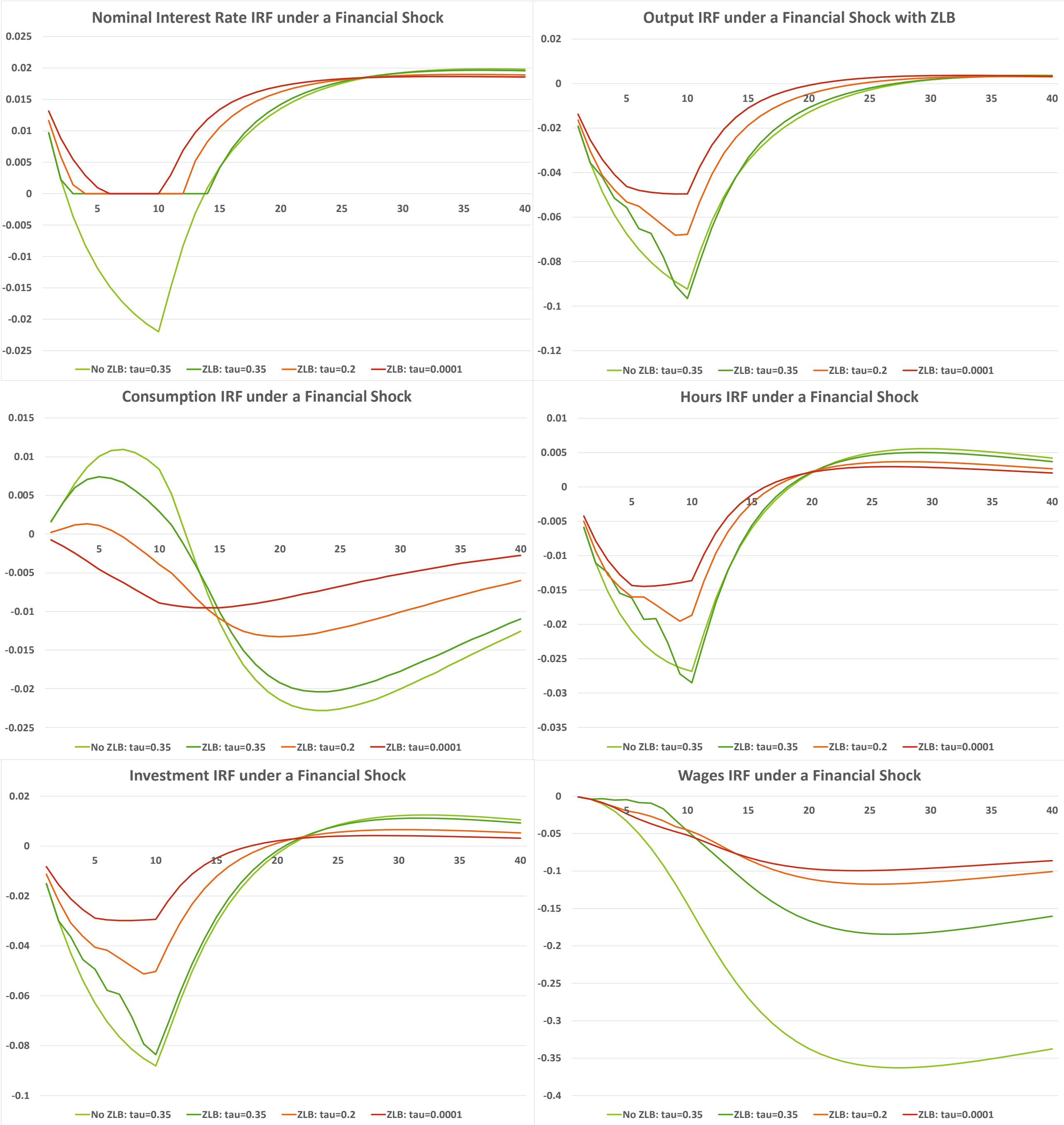


Figure 3.3: IRFs to a Financial Shock

Notes: Time, measured in quarters, in represented on the horizontal axis. Output, consumption, investment, hours and wages are measured in log deviations from steady state (in percentage). The nominal interest rate, the real interest rate and inflation are measured in percentage points deviations from the steady state, in annualized levels.

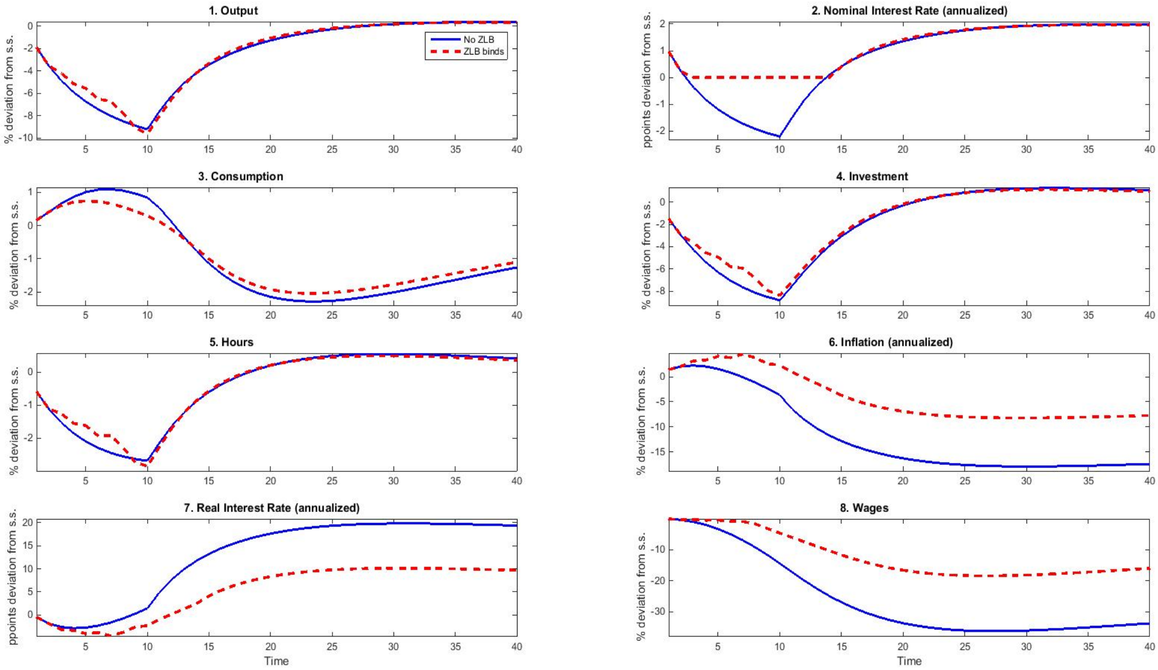


Figure 3.4: IRFs to a Financial Shock - $\tau = 0.35$

3.4.2.3 Effect of a Technological Shock

Considered one of the major driving forces behind economic fluctuations in most early standard New Keynesian DSGE models, the technological or productivity shock is usually used as the default shock to simulate when new frictions are incorporated into the model and need to be tested. However, several studies (including Smets and Wouters (2007), Jermann and Quadrini (2012), Lindé et al. (2016), just to name a few) that introduced new types of shocks into more traditional frameworks concluded that, although the productivity shock plays an important role in generating business cycles, it is not a dominant role since other shocks account for a higher contribution of the forecast error variance of output and other endogenous variables. In this case, we observe from Table 3.2 that the weight of the productivity shock to ouptut volatility is small (11.27%), although is the second largest responsible for movements in output. This contribution was even smaller for Smets and Wouters (2007) (4.1%) and Pfeifer (2016) (5.99%). However, due to its importance to the dynamics of the model, especially in the interaction with the price and wage rigidities and also with the enforcement constraint, it is imperative to analyse the dynamic path of the economy when a technological shock strikes.

3.4.2.3.1 The impact of the interest rate subsidy (τ) over the dynamic path of the variables

Figure 3.5 presents the impulse response of the nominal interest rate, output, consumption, hours, investment, and wages to a positive technological shock, simulated for the benchmark economy with no ZLB ($\tau = 0.35$) and for the economy with a binding ZLB and the 4 different values for the interest rate tax benefit ($\tau = 0.0001$, $\tau = 0.2$, $\tau = 0.35$ and $\tau = 0.5$).

Output rises for all the simulated cases, although in the moment of impact initially decreases slightly, bust immediately recovers and increases steadily even after the shock fades away. Investment also rises, although for lower values of the interest rate benefit ($\tau = 0.2$ and $\tau = 0.0001$) the initial impact is also slightly negative, but like output, it also recovers quickly to an ascending trajectory. The higher the interest rate tax benefit (τ) is, the greater is the incentive for firms to finance working capital through debt rather than equity (regardless of the presence of the presence of equity payout costs), and also the greater is the incentive for firms to increase its investment levels today to accumulate more capital in the future, which in turns allows firms to borrow more in the future to finance working capital and consequently, relax the tightness of the enforcement constraint in the future.

Hours, nominal wages and the nominal interest rate decline, while the real interest rate and inflation (as can be observed in Figure 3.6) also increase. Although a fall in hours might seem counter intuitive with a positive technological shock, it is consistent with the findings of Jermann and Quadrini (2012), since they show that their model with only productivity shocks does not generate enough volatility of hours, even with an alternative specification of preferences based on indivisible labor. This result is related to the fact that, in their model, working capital financing determines that the intra-period loan is defined by the production function, $l_t = F(z_t, n_t, k_t) = z_t k_t^{\theta} n_t^{1-\theta}$, which in turn affects directly the enforcement constraint $\xi_t(k_{t+1} - b_{t+1}/(1 + r_t)) \geq l_t$. Since $l_t = z_t k_t^{\theta} n_t^{1-\theta}$, it follows that a positive productivity shock makes the enforcement constraint tighter, requiring firms to increase its equity payout d_t and reducing the new intertemporal debt b_{t+1}. However, if firms are unable to exchange flexibly equity for debt due to rigidities (the equity payout cost), then firms must cut employment, and n_t decreases. Furthermore, according to Smets and Wouters (2007) "there has been a lively debate about the effects of productivity shocks on hours worked and about the implications of this finding for the role of those shocks in US business cycles." Smets and Wouters (2007) argue that several authors, including Galí (1999), Galí and Rabanal (2005) and Francis and Ramey (2005), have concluded that a positive productivity shock leads to an immediate fall in hours worked, due to several causes that include the presence of nominal price rigidities (prices and wages), habit formation, adjustment costs in investment, capital utilization costs, among many others.

For higher values of the interest rate subsidy ($\tau = 0.35$ and $\tau = 0.5$), the impulse responses are wider for almost all variables, specially for the nominal interest rate and investment. However, the duration of the ZLB is longer for lower values of τ: the ZLB remains binding for 11 quarters in the economy with $\tau = 0.0001$, while for $\tau = 0.5$, the spell of the ZLB is only 5 quarters. In this case, having a binding ZLB or not does not produce substantial differences in the responses of all variables, for $\tau = 0.35$.

The dynamic path of consumption, once again, fluctuates considerably depending on the value of the tax benefit chosen to perform the simulation: for higher values of τ ($\tau = 0.35$ and $\tau = 0.5$), consumption follows a decreasing trajectory immediately after the technological shock hits the economy, and for lower values of the interest rate subsidy ($\tau = 0.0001$, $\tau = 0.2$), that response is positive and wider. This finding holds whether the zero lower bound is binding or not. Once again, just as in the financial shock case, this behavior can be justified by the high persistence of the Taylor rule shock, the presence of price and wage rigidities and the fact that when $\tau \to 0$, the economy gets similar to a standard New Keynesian model with complete markets and and households adjust consumption to accommodate larger output and investment levels.

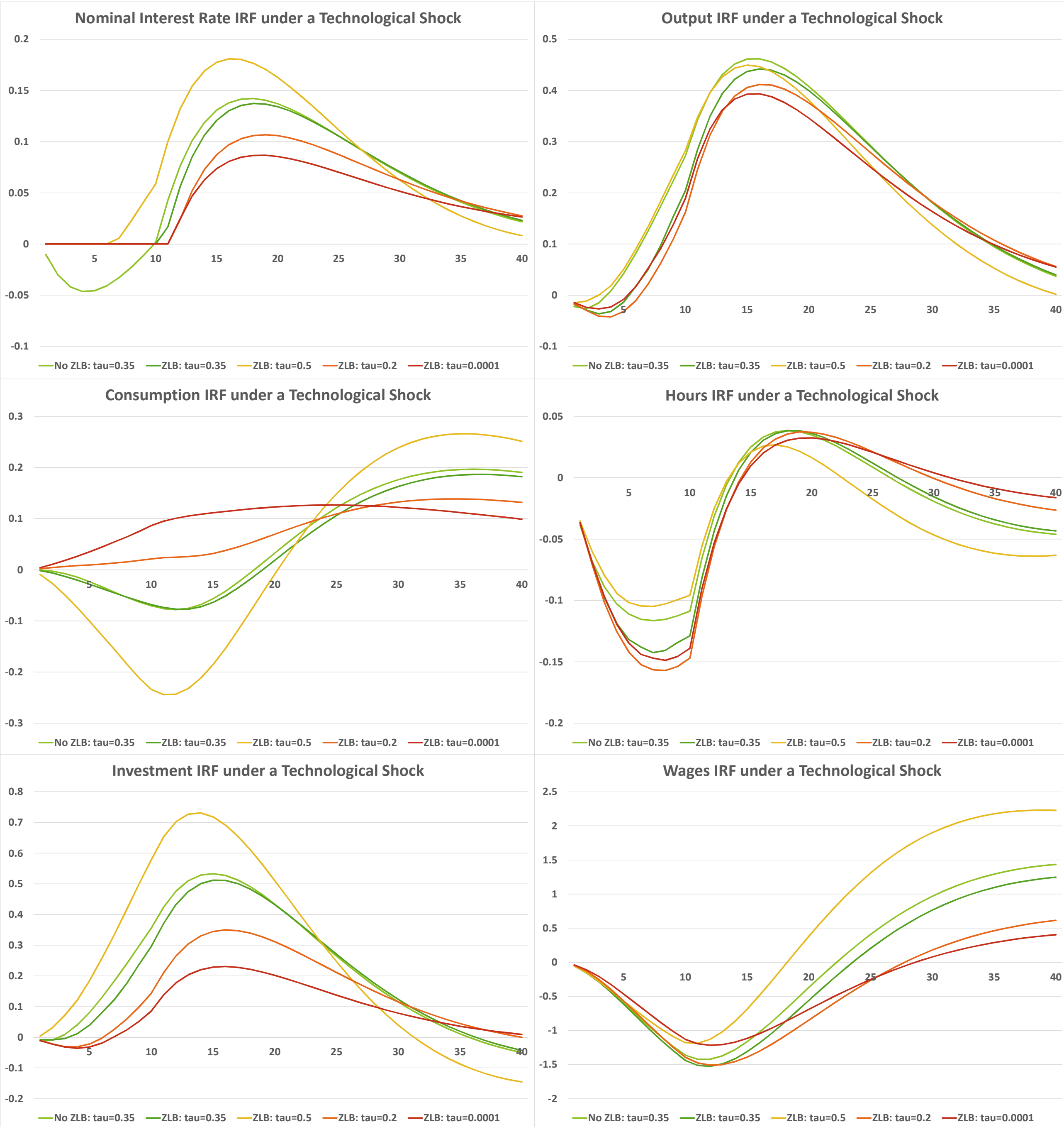


Figure 3.5: IRFs to a Technological Shock

Notes: Time, measured in quarters, in represented on the horizontal axis. Output, consumption, investment, hours and wages are measured in log deviations from steady state (in percentage). The nominal interest rate, the real interest rate and inflation are measured in percentage points deviations from the steady state, in annualized levels.

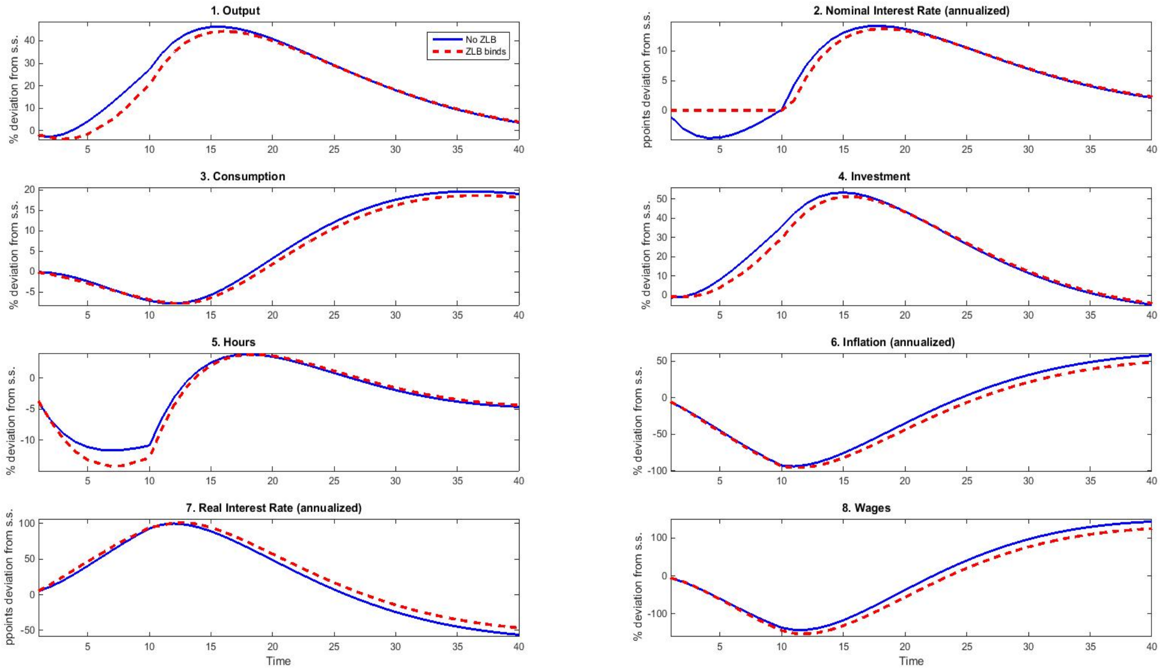


Figure 3.6: IRFs to a Technological Shock - $\tau = 0.35$

3.4.2.4 Effect of an Investment Shock

The investment shock, or a shock to the marginal efficiency of investment (MEI), is especially important to justify fluctuations in consumption and investment, as we can observe in Table 3.2. A positive investment shock makes the economy more productive at transforming investment into new physical capital. It can be considered, in a way, analogous to the technology shock, z_t, in the sense that makes the economy more productive at transforming capital and labor into output. In this case, an increase in ζ_t implies that firms can transform more capital goods into new capital goods K_{t+1} for a given amount of investment I_t, i.e. this shock increases the efficiency of investment. Some authors such as Justiniano et al. (2011) have argued that this shock is a reduced form proxy for modelling the health of the financial system, in the sense that the financial system indirectly converts investment into capital, and hence the higher (or lower) ζ_t, the better (or worse) the financial system is.

3.4.2.4.1 The impact of the interest rate subsidy (τ) over the dynamic path of the variables

The impulse responses for a 6 standard deviations negative investment shock, simulated for the benchmark economy (with $\tau = 0.35$, and the economy with the binding ZLB for four different values of the interest rate tax subsidy ($\tau = 0.0001$, $\tau = 0.2$, $\tau = 0.35$ and $\tau = 0.5$) are presented below in Figure 3.7. We observe that, for most simulations, a negative investment shock leads to a sizeable decrease in output, hours, investment and the real interest rate, and drops consumption, the nominal interest rate and inflation. The intuition behind this behavior is the following: when ζ_t decreases, transforming investment into capital units becomes less efficient, so agents decide to save less through capital, increasing consumption. In the job market, assuming that the labor demand curve is stable when the shock hits the economy, this leads to an inward shift of the labor supply curve, which initially lowers the number of hours worked and increases nominal wages. The negative response of labor consequently results in a reduction of output, which combined with the increase of consumption implies that investment also decreases. However, the reduction in investment is immediate since it is directly affected by the shock, ζ_t. As the initial impact of the shock fades away, the economy starts to accumulate less capital and the levels of consumption start to drop gradually, which shifts labor supply back out, increasing hours worked and dropping wages once more.

The exception for this trajectory of the impulse responses is the economy where a binding ZLB and with the highest tax benefit ($\tau = 0.5$): for this type and size of shock, this value of the interest rate subsidy is high enough to trigger anomalous behaviors in the dynamic path of the main endogenous variables of this model. In this case, although the nominal interest rate hits the zero lower bound after five quarters, there is a sudden sharp positive peak at period nine, followed by a steep fall at period ten, and after by a new upturn inversion, converging to the dynamic paths followed by the nominal interest rate in the other simulations. This anomalous peak is reflected in the responses of output, hours, investment, and in a smaller scale, in nominal wages. Although the dynamic path of consumption when $\tau = 0.5$ does not register the sudden peak, its response is the highest among the other simulations.

Figure 3.7: IRFs to an Investment (MEI) Shock

Notes: Time, measured in quarters, in represented on the horizontal axis. Output, consumption, investment, hours and wages are measured in log deviations from steady state (in percentage). The nominal interest rate, the real interest rate and inflation are measured in percentage points deviations from the steady state, in annualized levels.

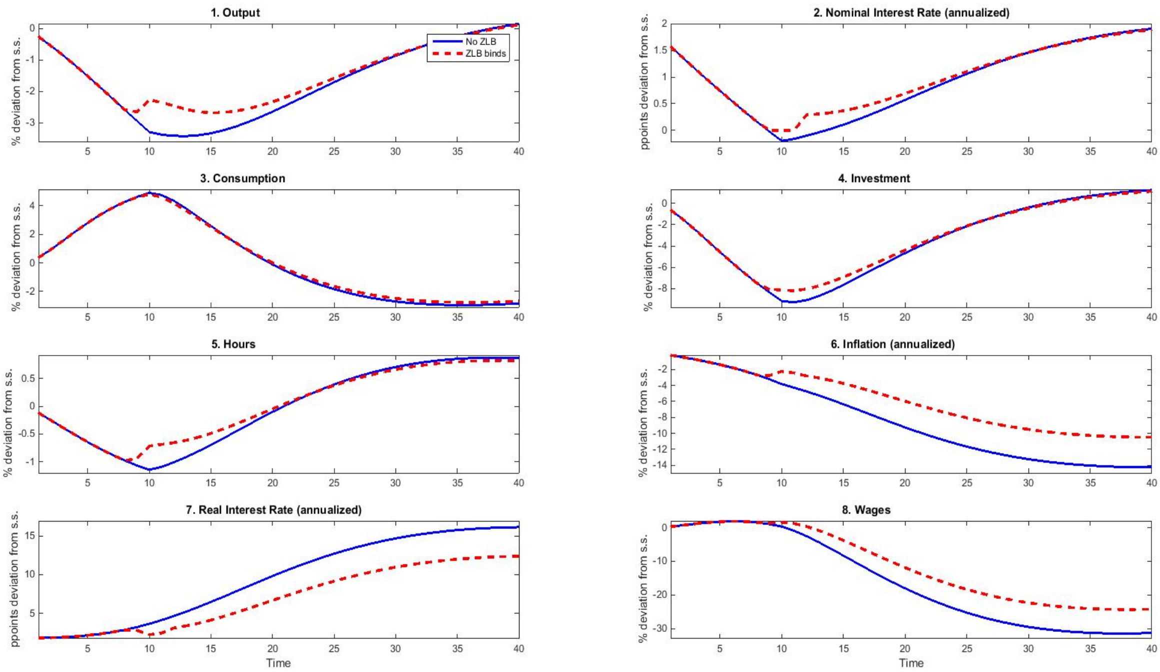


Figure 3.8: IRFs to a Investment (MEI) Shock - $\tau = 0.35$

3.4.2.5 Effect of a Price Markup Shock

The analysis of the price markup shock is particularly important, since according to the variance decomposition findings presented in Table 3.2, this shock is the major driving force for the volatility of output (46.26%) and the other major variables of the model, except for wages (15.54%) and inflation (15.34%).

3.4.2.5.1 The impact of the interest rate subsidy (τ) over the dynamic path of the variables

Figure 3.9 displays the impulse response functions to a positive 2 standard deviation price markup shock in the benchmark economy with no ZLB ($\tau = 0.35$) and for the economy with a binding ZLB and the 4 different values for the interest rate tax benefit ($\tau = 0.0001$, $\tau = 0.2$, $\tau = 0.35$ and $\tau = 0.5$). The price markup shock causes output, hours and investment to decline together, providing positive co-movement to their dynamic paths. However, the responses of consumption are much more mixed, showing that the response of consumption during the initial impact of the shock increases with the value of τ, for the economy with a binding ZLB. As $\tau \to 0$, the response of consumption becomes negative, which is theoretically the expected standard reaction to this type of shock. However, this atypical behavior is not due to the presence of the zero lower bound since in the economy where the ZLB is not binding and $\tau = 0.35$ consumption immediately increases after the shock hits the economy. A possible explanation to this puzzling trajectory of consumption can be explained by the interaction between the price markup shock and the enforcement constraint. Jermann and Quadrini (2012) argue that a positive price markup shock gives additional market power for firms, that will provide an incentive to reduce production levels to maximize profits. In turn, a lower production decreases the right hand side of the enforcement constraint $\xi_t \left(k_{t+1} - \frac{b_{t+1}}{P(1+r_t)} \right) \geq F(z_t, k_t, n_t)$, therefore reducing the tightness of this constraint. This allows firms to raise debt and pay more dividends, therefore increasing the incentive to consume more.

Furthermore, according to Fratto and Uhlig (2014), price markup shocks played an important role to avoid a even larger fall in inflation during the 2008 financial crisis and in the slow recovery of employment during the post-crisis period. Fratto and Uhlig (2014) also used the well-known pre-crisis Smets and Wouters (2007) model as the benchmark framework upon they added financial frictions and reestimated with post-crisis data, although their analysis focused essentially in explaining the puzzling combination of low employment and stable and positive inflation that characterized the period following the Great Recession.

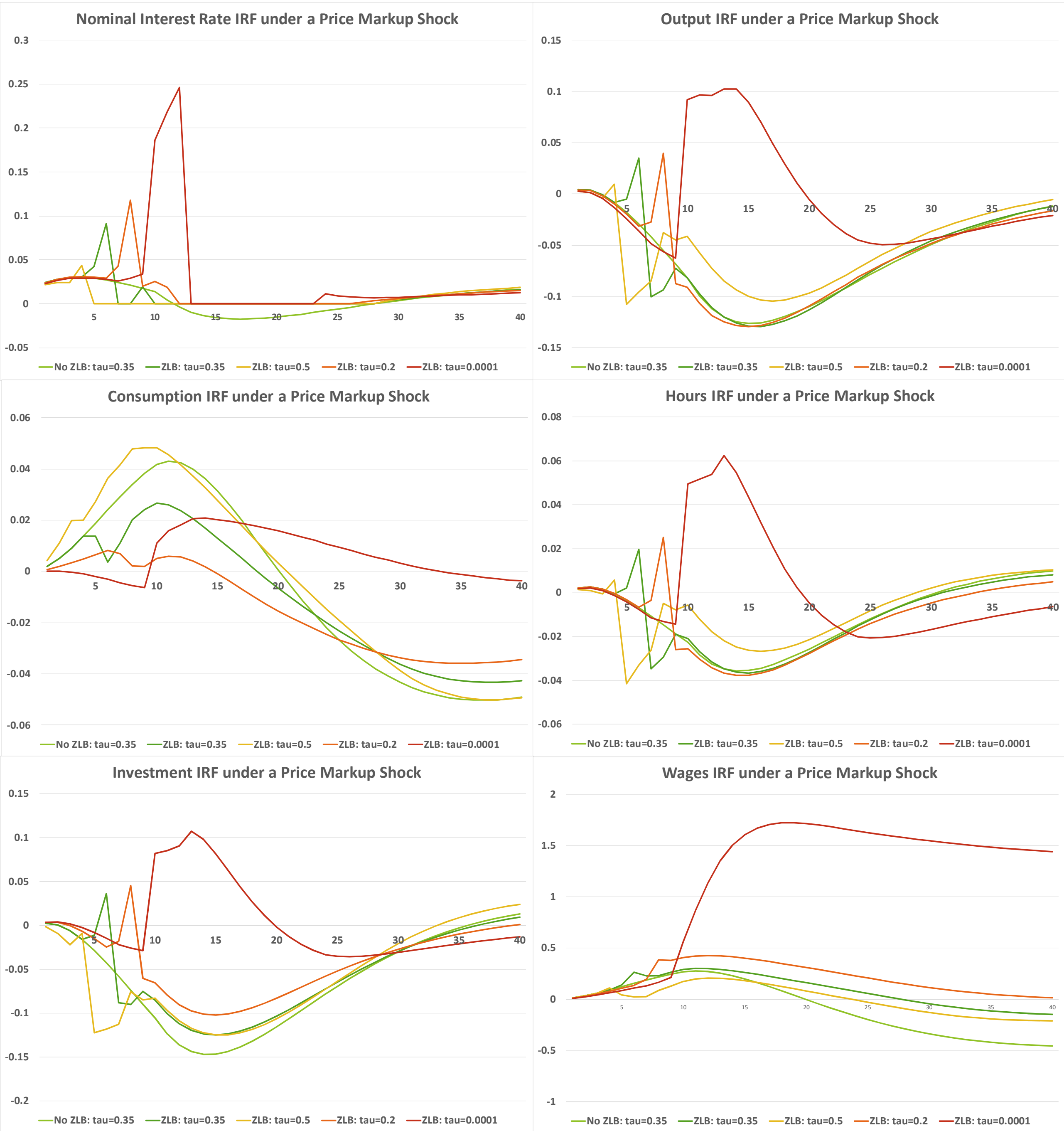


Figure 3.9: IRFs to a Price Markup Shock

Notes: Time, measured in quarters, in represented on the horizontal axis. Output, consumption, investment, hours and wages are measured in log deviations from steady state (in percentage). The nominal interest rate, the real interest rate and inflation are measured in percentage points deviations from the steady state, in annualized levels.

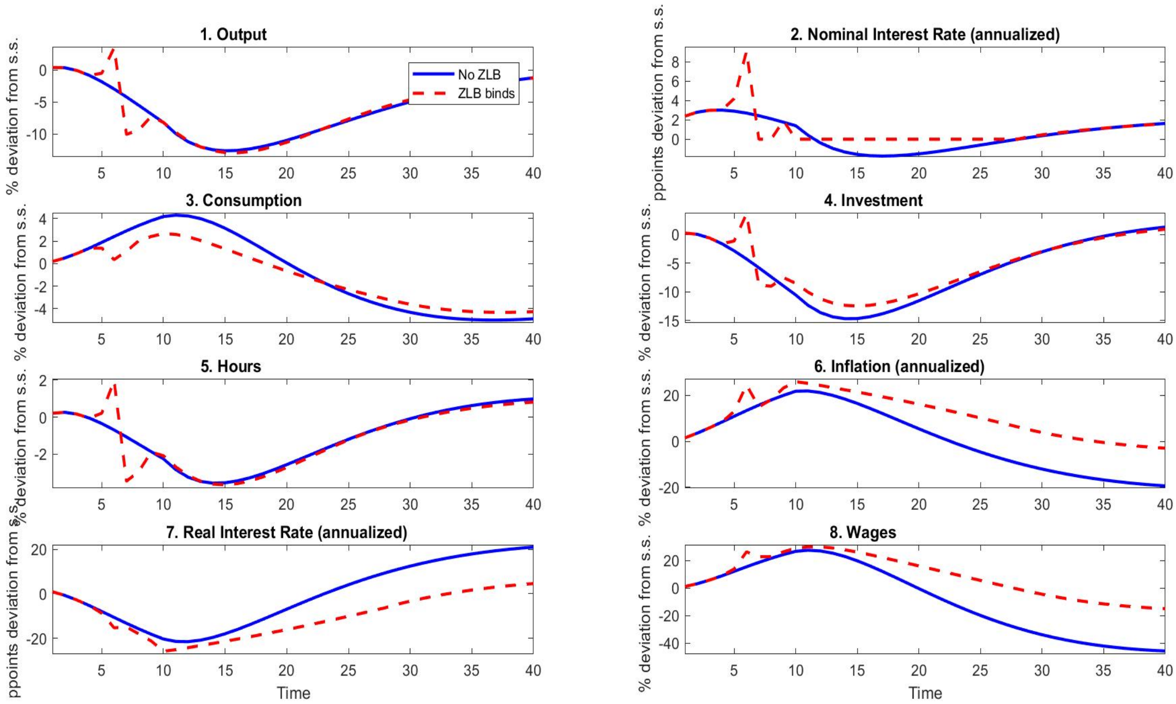


Figure 3.10: IRFs to a Price Markup Shock - $\tau = 0.35$

3.4.2.6 Effect of a Wage Markup Shock

Table 3.2 indicates that the contribution of the wage markup shock to the volatility of output is small (7.87%) [17], and it is also modest for the other variables of the model, although reaches a weight of 13.66% to the volatility of hours worked and 11.42% to movements of the nominal interest rate.

3.4.2.6.1 The impact of the interest rate subsidy (τ) over the dynamic path of the variables

Figure 3.11 displays the impulse responses to a 2 standard deviation positive wage markup shock, in the benchmark economy with no binding ZLB ($\tau = 0.35$) and for the economy with a binding ZLB, considering 4 different values for the interest rate tax benefit ($\tau = 0.0001$, $\tau = 0.2$, $\tau = 0.35$ and $\tau = 0.5$). Just like

[17] This contribution was also found to be small for Jermann and Quadrini (2012) (12.9%) and for Pfeifer (2016) (11.63%).

the price markup shock, this shock also induce extremely volatile responses for all variables, specially in the simulated economies with high levels of the interest rate benefit ($\tau = 0.35$ and $\tau = 0.5$). Overall, output falls after the shock hits, as well as hours, investment and the nominal interest rate. This is consistent with the findings of Smets and Wouters (2007), that also obtained a falling output (around 0.8 percent) and hours worked (around 0.6 percent), and also got initial positive but falling paths for both the nominal interest rate and inflation.

Wages increase, and consumption also increase, but only for higher levels of the interest rate subsidy, τ. It is important to emphasize, however, that for those higher values of τ, the responses of almost all variables exhibit very atypical behaviors, since those responses hit a positive peak before the shock fades away in period 10. In fact, due to that peak, the ZLB only binds in period 8 when $\tau = 0.5$, in period 10 when $\tau = 0.35$, and in period 13 for $\tau = 0.2$. In the economy with $\tau = 0.0001$, the ZLB does not even binds at any point of the simulation horizon. The duration of the ZLB also increase with the size of τ: it lasts for 12 quarters when $\tau = 0.5$ and when $\tau = 0.35$, but lasts only for 5 quarters when $\tau = 0.2$.

The smoothest responses correspond to the economy without a binding ZLB and to the economy with a binding ZLB but with an almost absent interest rate subsidy ($\tau = 0.0001$). For this two cases, the dynamic path of the nominal interest rate is very similar, characterized by a hump shape (there is a positive initial response to the shock, followed by a decreasing trajectory, until it turns upwards again from period 15 onwards). However, only the benchmark economy reaches negative values, since it is unconstrained by the ZLB. For output and hours worked, the responses of these two economies are almost coincident, and to a lesser extent, also the responses of nominal wages and investment. Once again, the exception is consumption, and in this case in the simulation without a binding ZLB there is a positive dynamic path for consumption, while for the constrained economy with almost no tax benefit ($\tau = 0.0001$) the response is negative. Once again this is related to the fact that with $\tau \to 0$ the economy is closer to a benchmark New Keynesian economy with complete markets, and also to the fact that a high persistent Taylor rule causes this type of distortions in the model.

From Figure 3.12 it is also possible to observe that the increase in wage markup puts upward pressure on inflation, but since that increase exceeds the initial increase in the nominal interest rate, the real interest rate drops after the shock hits, which provides an incentive to an increase in consumption.

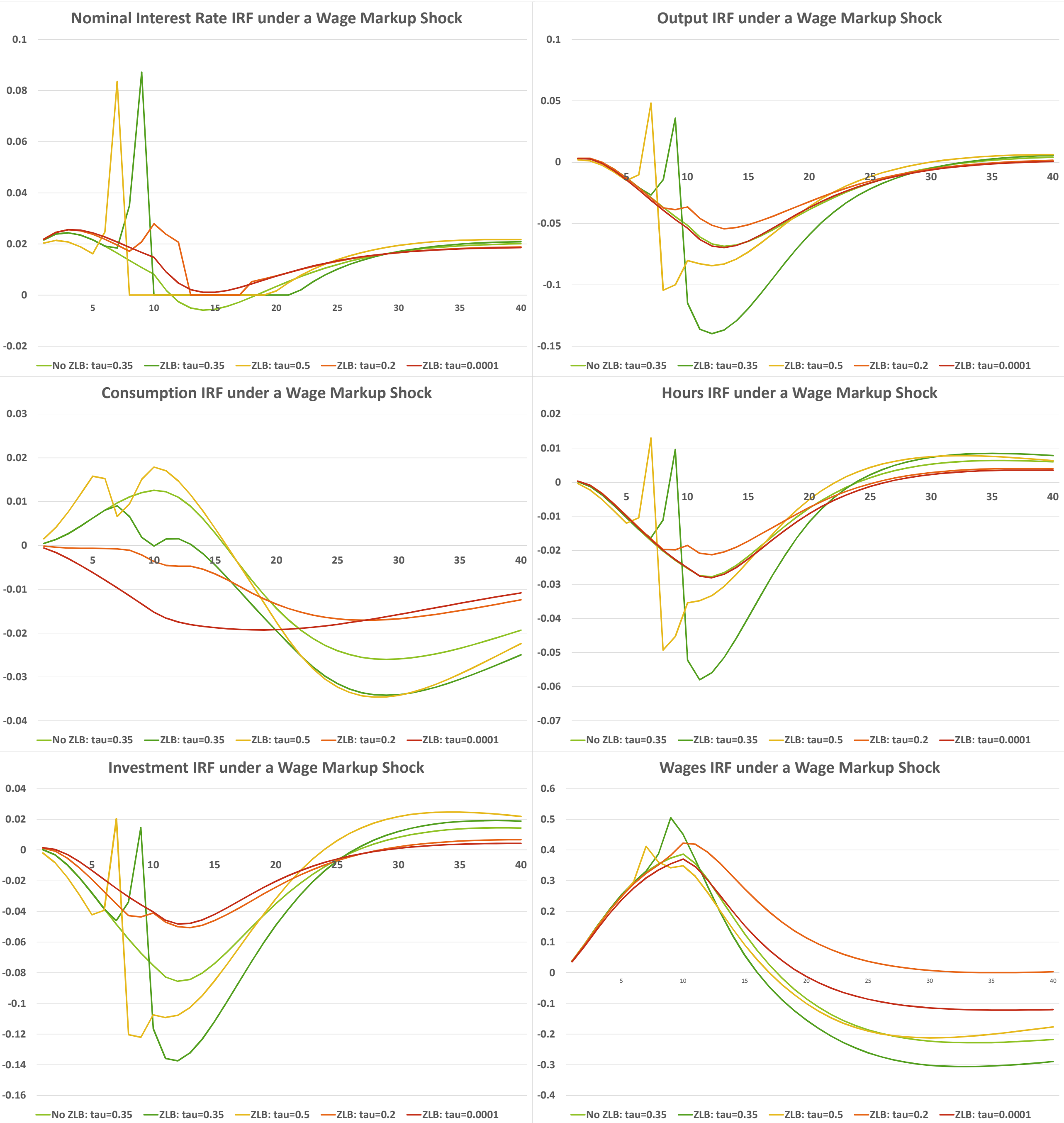


Figure 3.11: IRFs to a Wage Markup Shock with a binding ZLB

Notes: Time, measured in quarters, in represented on the horizontal axis. Output, consumption, investment, hours and wages are measured in log deviations from steady state (in percentage). The nominal interest rate, the real interest rate and inflation are measured in percentage points deviations from the steady state, in annualized levels.

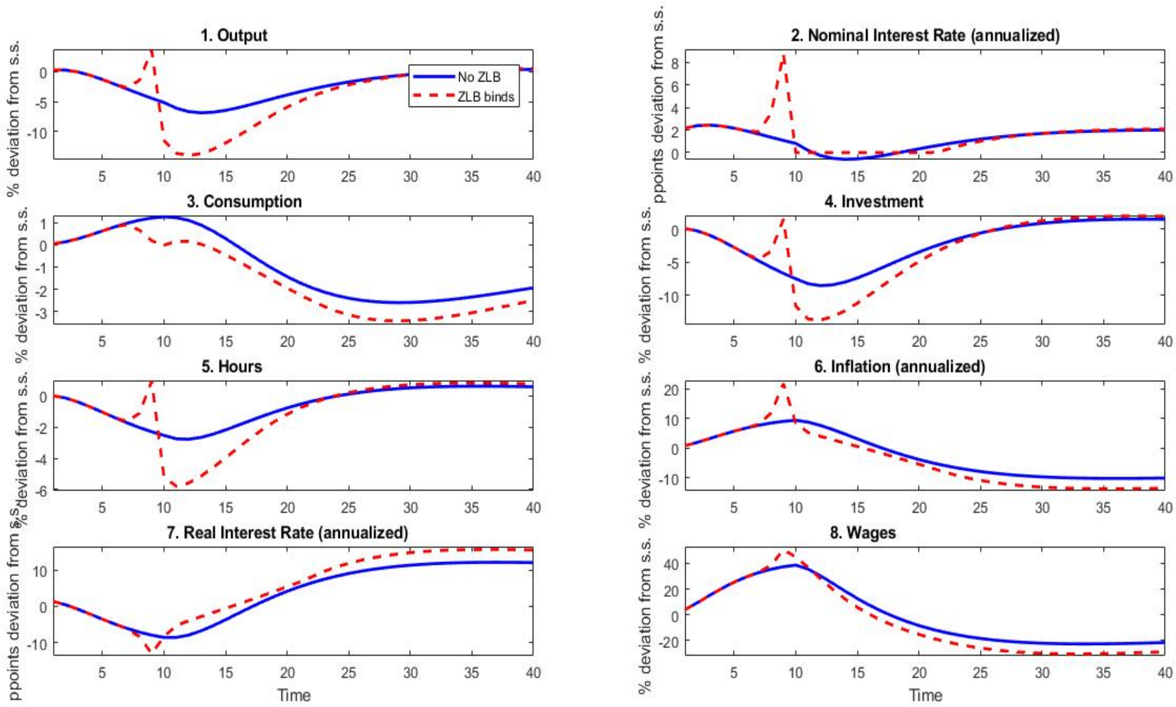


Figure 3.12: IRFs to a Wage Markup Shock - $\tau = 0.35$

3.4.2.7 Effect of a Government Expenditure Shock

According to Table 3.2, the government shock only accounts for about 5.2% of the total volatility of output, which is consistent with the estimated contribution of 5.96% computed by Pfeifer (2016), and still is bigger than the value of 0.8% reported by Jermann and Quadrini (2012). However, many authors such as Adolfsen (2017) concluded that, when the ZLB binds in a liquidity trap, government spending becomes more effective in stimulating output, and consequently the size of the government multiplier increases at the ZLB. In the next section we will explore how that relationship between a government expenditure shock and the ZLB is affected by the presence of the interest rate subsidy, τ.

3.4.2.7.1 The impact of the interest rate subsidy (τ) over the dynamic path of the variables

Figure 3.13 plots the impulse response functions to a positive government expenditure shock in the benchmark economy with no binding ZLB and $\tau = 0.35$ and for the economy with a binding ZLB, considering 4 different values for the interest rate tax benefit ($\tau = 0.0001$, $\tau = 0.2$, $\tau = 0.35$ and $\tau = 0.5$). Overall, output grows after along the simulation horizon, although immediately after the initial impact there is a negative response in the constrained economies, specially when $\tau = 0.35$ (in that case the negative response persists until period 10, when the shock is over). It is also for this simulation that the duration of the ZLB is longer (11 periods), followed by the constrained economy where $\tau = 0.5$ and 8 periods (2 years) when $\tau = 0.2$. In the constrained economy with the lowest simulated tax benefit ($\tau = 0.0001$), the ZLB spell is only of 2 quarters, and this is the case for which consumption starts to increase almost immediately after the shock hits. In the unconstrained economy with $\tau = 0.35$, there is also small initial negative response of consumption, but followed by a quick recover in period 6. With a positive government spending shock, households anticipate an increase in taxes in the future, and therefore lower consumption, as can be observed for all the simulated cases, and raise labor supply, which justifies the positive response of hours worked for all the economies considered. In addition, the initial negative response of output is perceived as lower income for households, which leads them to lower investment, as can be observed in the dynamic path of investment for all the simulated economies. In turn, the positive response of hours worked puts downward pressure on nominal wages and inflation, as can be observed in Figure 3.14. Since the negative response of inflation is bigger than the negative response of the nominal interest rate, the real interest rate increases, crowding out consumption and investment.

In this case, the ZLB contributes negatively to the effectiveness of the government expenditure shock, since output increases more in the benchmark case then in the economy with a binding ZLB (see Figure 3.14). The presence of the interest rate subsidy increases that effectiveness of government spending since the magnitude of the impulse response of output increases with a higher value of τ. In fact, when $\tau = 0.5$, the positive path of output even surpasses the benchmark unconstrained economy. In this simulation, the impulse responses of all variables are larger than the benchmark economy and the other constrained simulations, for lower values of τ. All the responses are smaller when the tax benefit is almost absent ($\tau = 0.0001$), nearly resembling the standard findings that we would get in a New Keynesian DSGE model. This result implies that the presence of the financial frictions introduced by Jermann and Quadrini (2012) (the enforcement constraint and the equity payout cost) is an important source of distortions in this model, that, in interaction with the ZLB, is able to generate wider and more unconventional responses from the main variables of the model.

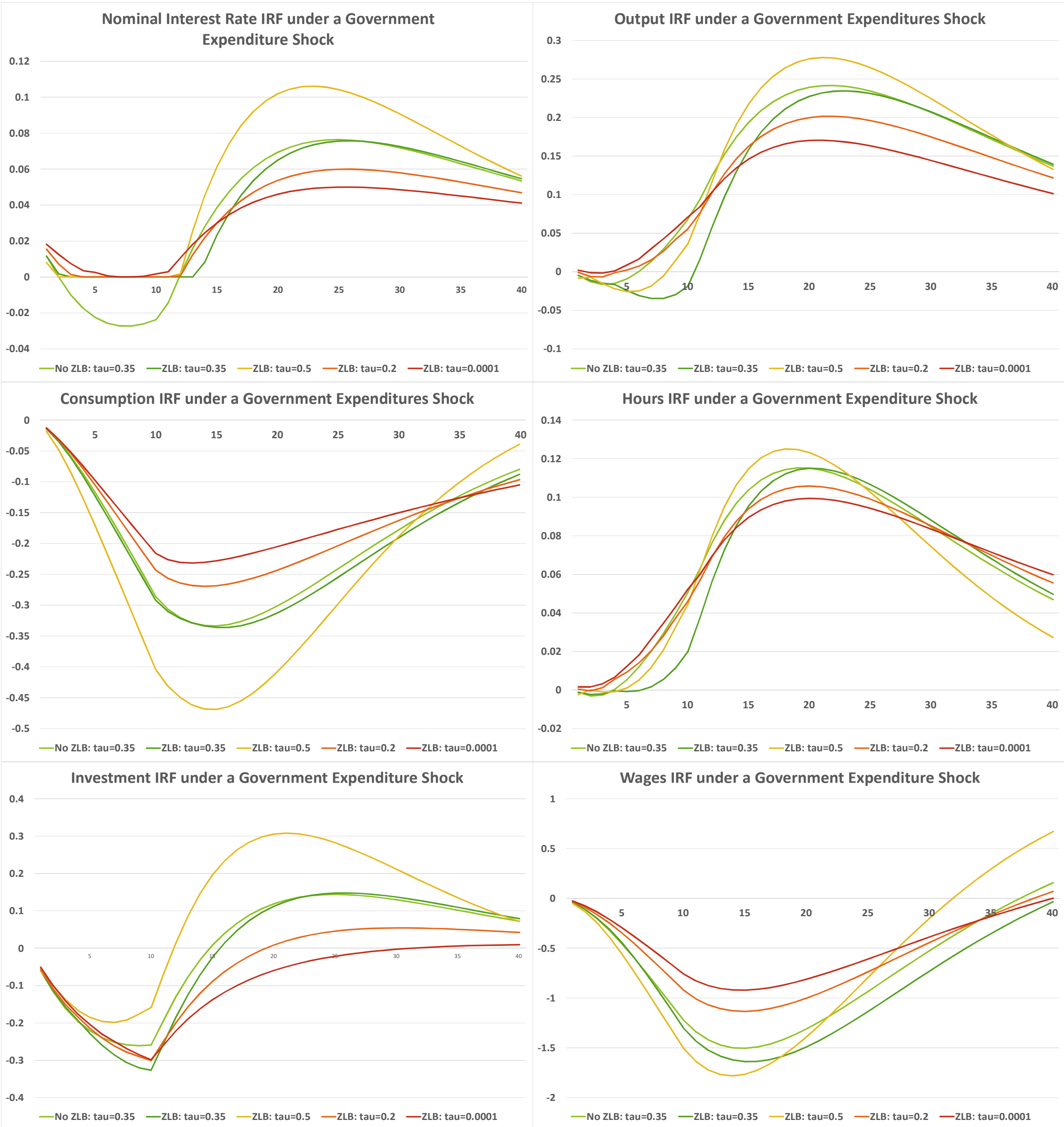


Figure 3.13: IRFs to a Government Expenditure Shock with a binding ZLB

Notes: Time, measured in quarters, in represented on the horizontal axis. Output, consumption, investment, hours and wages are measured in log deviations from steady state (in percentage). The nominal interest rate, the real interest rate and inflation are measured in percentage points deviations from the steady state, in annualized levels.

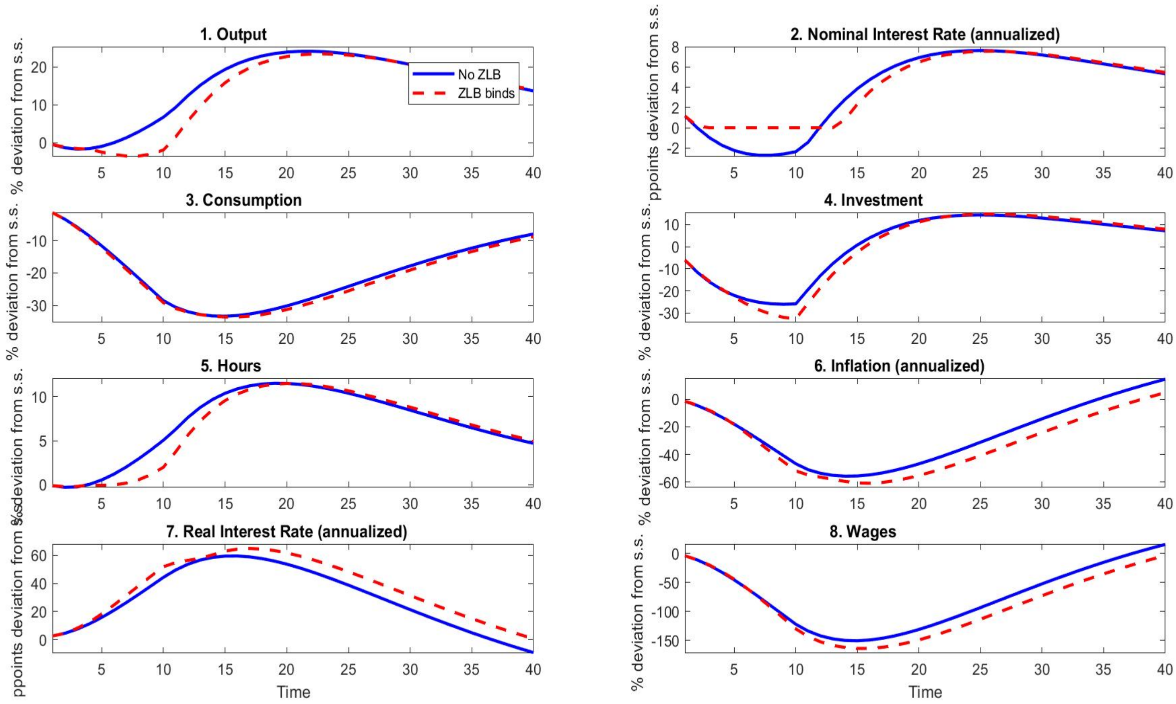


Figure 3.14: IRFs to a Government Spending Shock - $\tau = 0.35$

3.4.2.8 Effect of a Monetary Policy Shock

According to Table 3.2, the monetary policy shock only account for a small fraction of output, consumption, hours and investment developments, although it is particularly relevant to wages, inflation and debt repurchases volatility. Figure 3.15 shows the impulse response functions to a contractionary two standard deviation monetary policy shock that lasts for 10 quarters, for the five cases mentioned in the previous sections (the benchmark economy with no binding ZLB ($\tau = 0.35$) and the economy with a binding ZLB and the 4 different values for the interest rate tax benefit ($\tau = 0.0001$, $\tau = 0.2$, $\tau = 0.35$ and $\tau = 0.5$). Output, hours, investment and nominal wages all decline, while consumption, once again, exhibits different responses to this shock depending upon the value of the interest rate subsidy, τ. From Figure 3.16, which displays the impulse response functions to this shock but only for the benchmark economy and the economy with a binding ZLB when $\tau = 0.35$, we can also observe that in response to a monetary policy shock both the nominal and the real interest rates rise and inflation and wages decrease.

After the initial sudden increase, the nominal interest rate starts to gradually decline and, from Figure 3.15 it is clear that in the economy where the nominal interest rate is allowed to be negative, the paths for output, hours, wages and investment reach the lowest values among the different scenarios, except for consumption. In the constrained economy, the duration of the ZLB increases with the value of the tax benefit τ: for $\tau = 0.5$, the ZLB binds between quarters 5 to 12 while for $\tau = 0.2$ the ZLB only lasts for one period of the shock (10). When the interest rate subsidy is sufficiently close to zero ($\tau = 0.0001$), the ZLB does not bind, and the dynamic path of consumption is the only one that decreases after the monetary policy shock hits the economy, in accordance with the standard New Keynesian literature. That is also related with the fact that when $\tau = 0$, this model gets closer [18] to a standard New Keynesian model with complete markets. Rupert and Sustek (2019) argue that in a model with endogenous capital like this one, when output (income) temporarily drops as a response to a contractionary monetary policy shock in the presence of sticky prices, households can smooth consumption by reducing investment and adjusting the capital stock. Furthermore, Rupert and Sustek (2019) argue that in the cases of high persistence of the Taylor rule (which in this case is $\rho_R = 0.9$), consumption initially somewhat increases the higher ρ_R is, before declining below the steady state, as can be observed in Figure 3.15. Figure 3.15 also shows us that in the Jermann and Quadrini (2012) model, this initial temporary positive response of consumption increases with the value of the interest rate tax benefit, τ. With a higher tax subsidy τ, the duration of the ZLB increases, and that implies that the adjustment in investment to secure smooth consumption with a minimal effect on the real interest rate has to be stronger, therefore ensuring a smaller drop in output. If output/income falls less, that implies that consumption exhibits a larger positive response to a monetary policy shock when τ is higher.

These impulse responses to a monetary tightening in the form of a Taylor rule that increases the short term nominal interest rate are expected in the context of the standard New Keynesian monetary transmission mechanism related to this type

[18] In order to fully collapse to a standard New Keynesian model with complete markets, the equity payout cost must also be absent, i.e. $\kappa = 0$

of shock, based on the traditional interest rate channel that was vastly explored in the literature (e.g. Christiano et al. (1999), Christiano et al. (2005), Bernanke and Gertler (1995), Ireland (2016)). Since nominal prices (and consequently, inflation) are sluggish due to the Calvo rigidity that is imposed through a costly or staggered pricing scheme, it is expected that the increase in the nominal interest rates translates into an increase in the real interest rate, that in turn dampens consumption and investment. Through the Phillips curve, the decline in output puts downward pressure on inflation, which adjusts only gradually after the shock. However, due to the presence of both price and wage rigidities and also to the presence of the ZLB, this transmission mechanism in this framework does not operate so smoothly as the theoretical New Keynesian benchmark model would predict. Although both price and wage rigidity tend to amplify the responses of real variables to a monetary policy shock, both reactions are usually quite similar with either type of rigidity, since both the price and wage markups move in the same direction. As a consequence, when the contractionary monetary policy shock exert downward pressure on prices, some firms end up with prices that are too high relative to what they would optimally like, and therefore the price markup rises, and the same reasoning is valid for the wage markup.

However, in this particular model with the Jermann and Quadrini (2012) calibration, the values of the parameters associated with the Calvo wage adjustment and the Rotemberg price adjustment cost, respectively, are set low enough ($\omega = 0.278$ and $\phi = 0.031$) to not be considered sufficiently big to trigger more extreme responses from real variables. Therefore, in this case, the presence of the zero lower bound constraint is one of the most important sources of disparity of the impulse responses generated by this model.

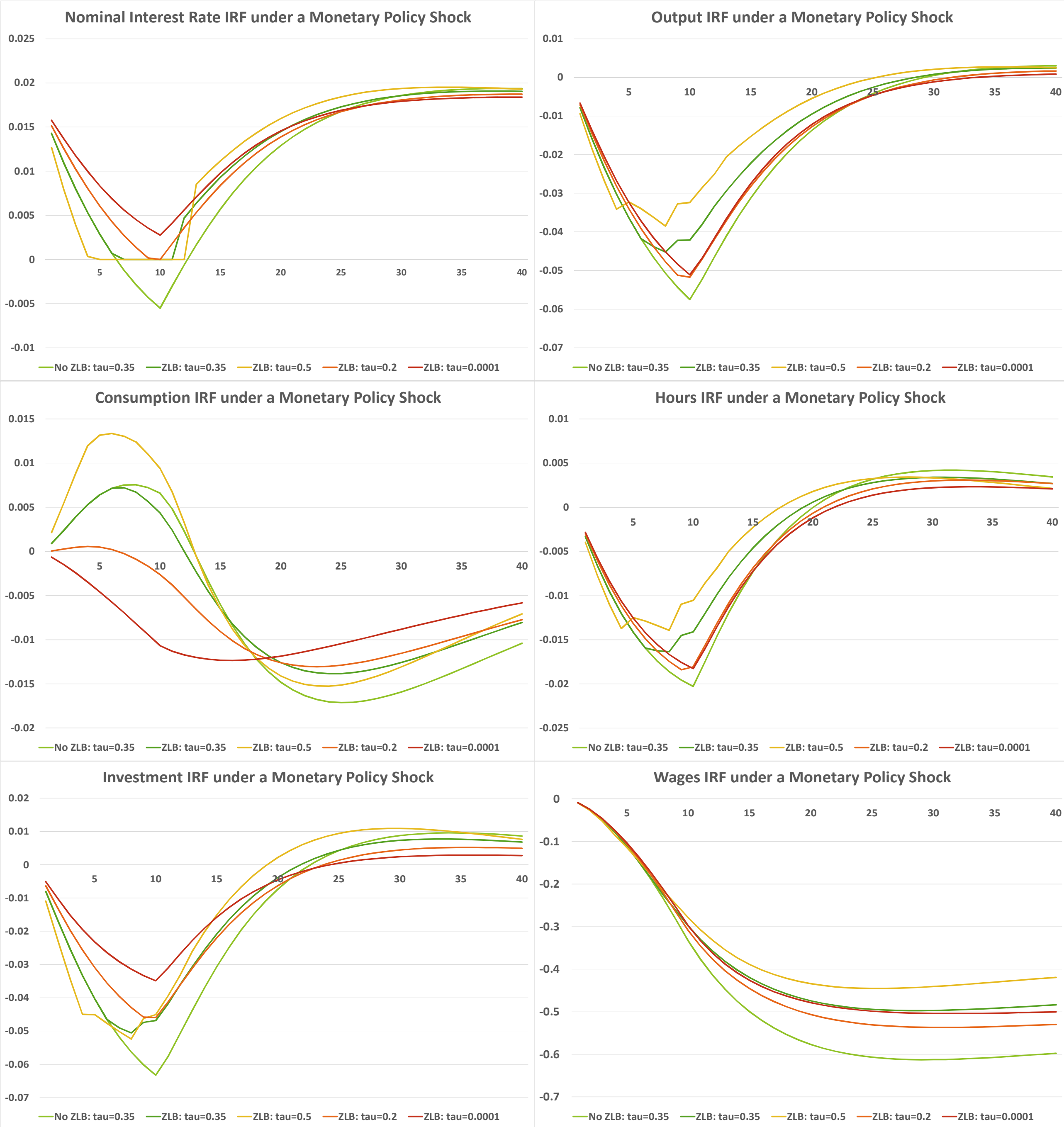


Figure 3.15: IRFs to a Monetary Policy Shock

Notes: Time, measured in quarters, in represented on the horizontal axis. Output, consumption, investment, hours and wages are measured in log deviations from steady state (in percentage). The nominal interest rate, the real interest rate and inflation are measured in percentage points deviations from the steady state, in annualized levels.

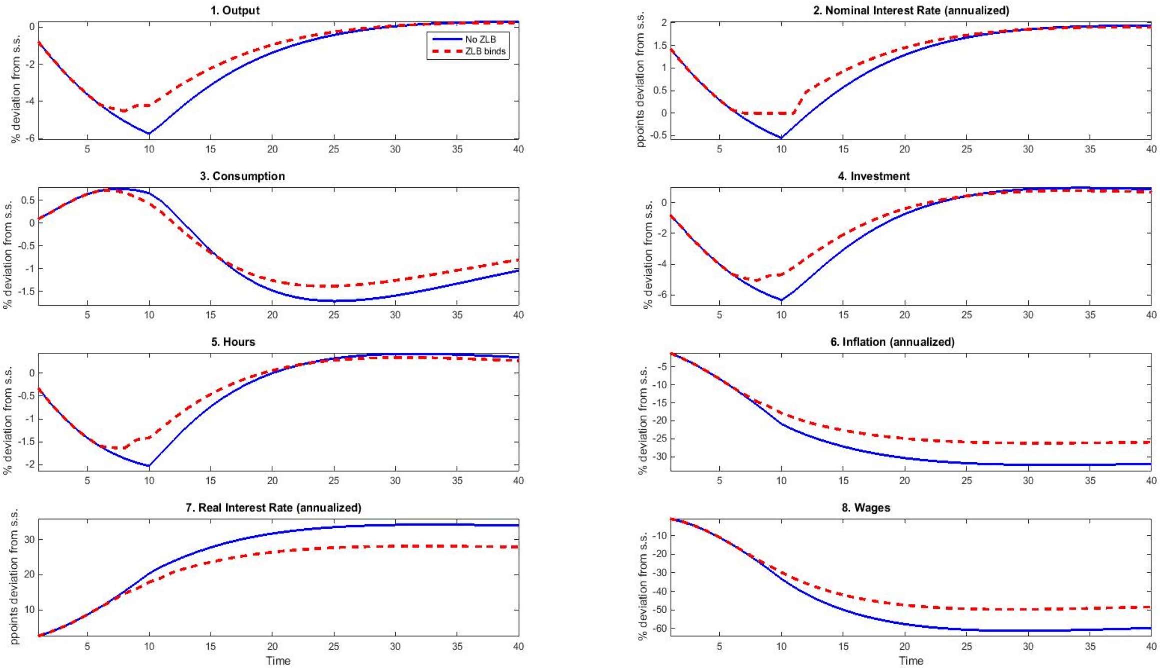


Figure 3.16: IRFs to a Monetary Policy Shock - $\tau = 0.35$

3.4.2.9 Statistical properties of the innovations and their relation to the endogenous variables

Table 3.3 presents an overview of the statistical properties of the most important endogenous variables of the model, which include GDP, consumption, investment, hours worked, the nominal interest rate, inflation rate, debt repurchases and equity payout to shareholders. Those statistical properties are measured by the mean, standard deviation, variance, skewness and kurtosis of those variables, all computed in a simulated economy with an interest rate subsidy of $\tau = 0.35$.

Surprisingly, the most volatile variables are wages, inflation and specially debt repurchases, when usually it is expected to get a much more unstable behavior for investment, for example. In general, all the variables seems to be low skewed, although some are more left-skewed (consumption, wages, inflation, debt repurchases and equity payout) and others are slightly right-skewed (GDP, investment, hours and the nominal interest rate. All the variables present very low values of kurtosis, suggesting that they are almost mesokurtic, although some exhibit a slight tendency to be more leptokurtic (hours, nominal interest rate and equity payout) and others to be more platykurtic and possess fatter tails (GDP, consumption, investment, wages and inflation).

Table 3.3: **Moments of Simulated Variables**

Variables	**Mean**	**Std.Dev.**	**Variance**	**Skewness**	**Kurtosis**
GDP	0.995	0.037	0.001	0.145	-0.484
Consumption	0.608	0.026	0.001	-0.150	-0.021
Investment	0.211	0.045	0.002	0.175	-0.322
Hours	0.299	0.012	0.0001	0.237	0.013
Wages	1.858	0.397	0.158	-0.395	-0.909
Interest Rate	0.018	0.013	0.0002	0.191	0.165
Inflation	0.027	0.217	0.047	-0.327	-1.065
Debt Repurchases	3.618	1.115	1.244	-0.416	-0.579
Equity Payout	0.204	0.037	0.001	-0.112	0.150

Source: Author's own calculations in Dynare.

As observed in Figure 3.4, the statistical properties of the 8 structural shocks included in the model somehow confirm the previous conclusions regarding the relative importance of each one as crucial drivers of recession episodes. The shocks with the highest absolute volatility are the wage markup shock and the price markup shock, followed by the preference shock. However, if we take into account to coefficient of variation to take into account the volatility of each innovation relative to its mean, the investment shock is the most volatile innovation, followed by the financial shock, the price markup shock and the preference shock. This is relatively consistent with the previous analysis, if we take into account that the shocks that needed to be simulated with a lower number of standard deviations were volatile enough to bind the ZLB and provide a wider duration of this constraint. Less responsive shocks, like the technological shock and the government expditure shock were simulated with a much higher nember of standard deviations in order to bind the ZLB and generate a sufficiently large duration of that constraint. Indeed, the wage markup shock, the price markup shock and the preference shock were simulated with 2 standard deviations, the financial shock with 4 standard deviations and the investment shock with 6 standard deviations. This is also consistent with the variance decomposition presented in Table 3.2, that suggest that the shocks that are more capable to generate more extreme movements in economic activity are the price markup shock, wage markeup shock and also the preference shock. The only exception in this case is the productivity shock, that, although exhibits a low volatility levels, contributes with approximately 11.27% to the volatility of output, the second largest driver of output movements according to these simulations.

Table 3.4: **Statistical Distribution of Innovations**

Innovations	**Mean**	**Std.Dev.**	**Variance**	**Coef. of Variation**	**Skewness**	**Kurtosis**
Preferences	0.9980	0.0446	0.0020	22.3844	-0.1159	-0.4474
Financial	0.1998	0.0055	0.00003	36.5201	0.1481	0.8313
Technology	0.0019	0.0121	0.0001	0.1587	-0.1375	-0.5469
Investment	1.0018	0.0142	0.0002	70.5372	0.1557	-0.3685
Price Markup	1.1398	0.0506	0.0026	22.5114	0.0171	-0.1454
Wage Markup	1.0216	0.1079	0.0116	9.4679	-0.1134	-0.0907
Gov. Spending	0.1776	0.0169	0.0003	10.5346	-0.0019	-0.3486
Monetary Policy	0.00003	0.0020	0.00004	0.0015	0.0202	-0.1891

Source: Author's own calculations in Dynare.

Another way to look at the strong linkages between some of the included structural shocks and endogenous variables, the correlations between the estimated innovations and the main variables of the model are presented in Table 3.5. The strongest correlation in this table (which exceeds 0. in absolute terms) is observed between equity payout and the price markup shock (0.6131), followed by the correlation observed between output and the price markup shock (-0.5008), which is consistent with the contribution of approximately 46.26% of this shock to output volatility. Investment is also highly correlated with the price markup shock (0.4795) and with the investment shock (0.4228), as well as hours worked (correlation of 0.3969 with the price markup shock and 0.2446 with the investment shock). The correlations between the preference shock and consumption (0.2425), investment (-0.3188), the nominal interest rate (0.1833) and debt repurchases (-0.2089) are also considerably strong, confirming the importance of the preference shock in driving dynamics into the economic activity. The correlations between the financial shock and wages (0.3298), inflation (0.3176) and debt repurchases (0.3842) are also considerably strong, confirming the link between the enforcement constraint and the marginal cost of labor, although the correlation between this shock and hours (0.0643) and between this shock and output (0.0703) is not high enough to reflect that relationship.

Table 3.5: **Correlation between Simulated Variables and Innovations**

	Variables								
Innovations	**y**	**c**	**i**	**n**	**w**	**r**	π	**b**	**d**
Preferences	-0.1546	0.2425	-0.3188	-0.0243	-0.057	0.1833	-0.0534	-0.2089	0.1569
Financial	0.0703	0.0408	0.0294	0.0643	0.3298	0.16	0.3176	0.3842	0.0603
Technology	0.1913	0.1526	0.1375	-0.31	-0.2628	-0.0333	-0.3072	-0.0914	0.008
Investment	0.2652	-0.2789	0.4228	0.2446	-0.0721	0.3127	-0.0513	0.129	0.0171
Price Markup	-0.5008	0.2137	-0.4795	-0.3969	0.3363	-0.1659	0.4002	0.1739	0.6131
Wage Markup	0.0396	0.0574	-0.0464	-0.0695	0.1135	0.1892	0.0467	0.0779	0.196
Gov. Spending	0.2195	-0.2366	-0.0223	0.1457	-0.3769	0.0249	-0.3856	-0.1699	-0.0392
Monetary Policy	-0.1084	-0.0223	-0.0657	-0.114	-0.0391	-0.1267	-0.0516	-0.0477	-0.1179

Source: Author's own calculations in Dynare.

3.5 Conclusion

In general, I conclude that, in an economy constrained by the binding presence of a zero lower bound imposed in a truncated Taylor rule by the monetary authority, the presence of an interest rate subsidy and other frictions (such as the equity payout cost, price and wages rigidities and investment adjustment costs) helps to amplify and propagate the effects of shocks of very different sources that can affect the economy.

While performing simulations of the model based on Jermann and Quadrini (2012), the unconstrained economy where the nominal interest rate is allowed to float freely (even to negative values) and with an interest rate tax benefit set to 35% ($\tau = 0.35$) was considered as the benchmark economy or the first best allocation, and compared against five different economies with a binding zero lower bound but different values of the tax benefit defined for each case: $\tau = 0.0001$, $\tau = 0.2$, $\tau = 0.35$ and $\tau = 0.5$. After conducting simulations for the eight different types of shocks included in the Jermann and Quadrini (2012) setting (preference, financial, productivity, investment, price markup, wage markup, government spending and monetary policy shocks), one of the first conclusions that stand out is the fact that the presence of a binding zero lower bound constraint is sufficient to trigger adverse shocks and increase the amplification and propagation of those shocks. My findings also suggest that using the interest rate subsidy as a fiscal instrument to circumvent completely the effects of a liquidity trap and replicate the first best allocation achieved in the benchmark economy is not possible, although it can help to neutralize partially the effects caused by that constraint.

For almost all simulated shocks, the magnitude of the impulse responses, which implies higher amplification and propagation of those shocks, increase with a higher interest rate subsidy, and for some values of the tax benefit (for $\tau > 0.35$) those responses usually surpass the dynamic path of the benchmark case. Furthermore, increasing the tax benefit as a fiscal tool to neutralize the effects of the ZLB further increases the duration of the ZLB, which contributes to perpetuate and reinforce the impact of the shocks over the economy and further cementing those shocks as important sources of economic fluctuations that are able to generate deep recessions like the 2008 financial crisis.

However, using lower values of the interest rate benefit or even completely eliminating this subsidy can approximate the economy to a standard New Keynesian economy with complete markets, which for some simulations implies that the zero lower bound can be completely circumvented (for example, for $\tau = 0.0001$ in the preference shock, the investment shock, the wage markup shock and the monetary policy shock simulations). However, the benchmark allocation can no longer be achieved, which implies that the positive impact over output will be lower and the capacity of the model to generate realistic business cycles will be limited.

These findings suggest that the tightening of the enforcement constraint imposed to firms during the 2008 financial crisis, limiting their ability to contract new debt to finance investment and labor, allied to the subsequent period of very low nominal interest rates contributed to extend and amplify the negative effects that different types of shocks exerted over the economy when they were triggered by the subprime crisis. Furthermore, the high persistence of the Taylor rule and the high values set to calibrate the output gap growth coefficient of the Taylor rule also help to justify the high persistence in the shocks processes that lead to a higher duration of the zero lower bound constraint and to explain the anomalous behavior of consumption in some of the simulations.

Appendix

3.5.1 Corrections in the First Order Conditions for the Firm's Problem

This section of the appendix describes the full set of equations that characterize the equilibrium of the structural model studied in this paper, presented by Jermann and Quadrini (2012) as an extension of the framework originally developed by Smets and Wouters (2007), by adding financial frictions and financial shocks. The full list of log-linearized equations is provided, as well as the the first order conditions of the firm, already incorporating the corrections proposed by Pfeifer (2016).

3.5.1.1 Dynamic System of Equilibrium Equations

This section presents the complete list of dynamic equations that define the equilibrium of the structural model. In the first order conditions for the firm the Lagrange multiplier λ is eliminated by imposing the condition $\lambda_t = 1/P_t \varphi_d(d_))$.

1. Households' Euler equation for bonds:

$$(1 + r_t) E m_{t+1} \frac{P_t}{P_{t+1}} - 1 = 0 \tag{3.31}$$

2. Capital utilization:

$$(1 - \mu_t \varphi_{d,t}) F_{u,t} - \Psi_{u,t} k_t - \chi_t D_{u,t} \varphi_{d,t} = 0 \tag{3.32}$$

3. Euler equation for capital:

$$E m_{t+1} \left\{ (1 - \delta) Q_{t+1} + \frac{F_{k,t+1} - \Psi_{t+1}}{\varphi_{d,t+1}} - \mu_{t+1} F_{k,t+1} - \chi_{t+1} D_{k,t+1} \right\} + \xi_t \mu_t - Q_t = 0 \tag{3.33}$$

4. Price of capital:

$$Q_t \Upsilon_{i,t} + E m_{t+1} Q_{t+1} \Upsilon_{i_{-1},t+1} - \frac{1}{\varphi_{d,t}} = 0 \tag{3.34}$$

5. Law of motion for capital:

$$(1 - \delta) k_t + \Upsilon_t - k_{t+1} = 0 \tag{3.35}$$

6. New wage (linearized):

$$-\left(\frac{h\sigma\Phi}{1-h}c_{t-1}\right)+\left(\frac{\sigma\Phi}{1-h}\right)+\Phi_t P_t+\Phi\upsilon_t+\frac{\Phi}{\varepsilon}n_t+\frac{\upsilon\Phi}{(\upsilon-1)\varepsilon}W_t+\beta\omega E_t w_{t+1}-w_t=0 \tag{3.36}$$

where $\Phi = \frac{\varepsilon(\upsilon-1)(1-\beta\omega)}{\varepsilon(\upsilon-1)+\upsilon}$.

7. Wage index:

$$\left[\omega W_{t-1}^{\frac{1}{1-\upsilon_t}} + (1-\omega)w_t^{\frac{1}{1-\upsilon_t}}\right]^{1-\upsilon_t} - W_t = 0 \tag{3.37}$$

8. Labor demand:

$$(1-\mu_t\varphi_{d,t})F_{n,t} - \frac{W_t}{P_t} - \chi_t\varphi_{d,t}D_{n,t} = 0 \tag{3.38}$$

9. Bond demand:

$$R_t E m_{t+1}\left(\frac{P_t\varphi_{d,t}}{P_{t+1}\varphi_{d,t+1}}\right) + \xi_t\mu_t\varphi_{d,t}\left(\frac{R_t}{1+r_t}\right) - 1 = 0 \tag{3.39}$$

10. Nominal price:

$$P_t\left[G_{2,t} + E m_{t+1}\left(\frac{\varphi_{d,t}}{\varphi_{d,t+1}}\right)G_{1,t+1}\right] - \chi_t\varphi_{d,t} = 0 \tag{3.40}$$

11. Firm's value:

$$d_t + E m_{t+1}V_{t+1} - V_t = 0 \tag{3.41}$$

12. Enforcement constraint:

$$\xi_t\left(k_{t+1} - \frac{b_{t+1}}{P_t(1+r_t)}\right) - F_t = 0 \tag{3.42}$$

13. Firm's budget:

$$P_t\Big[F_t - \Psi_t k_t\Big] + \frac{b_{t+1}}{R_t} - b_t - W_t n_t - P_t G_t - P_t\varphi_t - P_t i_t = 0 \tag{3.43}$$

14. Household's budget:

$$W_t n_t + P_t d_t - \frac{b_{t+1}}{1+r_t} + b_t - P_t c_t - T_t = 0 \tag{3.44}$$

15. Government budget:

$$P_t G_t + B_{t+1}\left(\frac{1}{R_t} - \frac{1}{1+r_t}\right) - T_t = 0 \tag{3.45}$$

16. Monetary policy (linearized):

$$a_1 r_{t-1} + a_2(P_t - P_{t-1}) + a_3(Y_t - Y_t^*) + a_4(Y_{t-1} - Y_{t-1}^*) + \varsigma_t - r_t = 0 \tag{3.46}$$

where $a_1 = \rho_R$, $a_2 = (1-\rho_R)\nu_1$, $a3 = (1-\rho_R)\nu_2 + \nu_3$, $a_4 = -\nu_3$.

17. Output:

$$F_t - Y_t = 0 \tag{3.47}$$

18. Debt repurchase:

$$\frac{b_t/(1+r_{t-1}) - b_{t+1}/(1+r_t)}{Y_t P_t} - x_t = 0 \tag{3.48}$$

Taking into account that $1 + r = (R - \tau)/(1 - \tau)$, we can use the linearized version of these equations to solve for 18 variables:

$R_t, P_t, c_t, n_t, u_t, d_t, \mu_t, \chi_t, x_t, Q_t, i_t, w_t, W_t, Y_t, V_t, T_t, k_{t+1}, b_{t+1}$

as a function of 16 states:

$z_t, \zeta_t, \gamma_t, \eta_t, \upsilon_t, G_t, \varsigma_t, \xi_t, p_{t-1}, i_{t-1}, c_{t-1}, W_{t-1}, R_{t-1}, Y_{t-1}, k_t, b_t.$

3.5.2 Variance Decomposition (in percent)

Table 3.6: Variance Decomposition (Original Jermann and Quadrini (2012) findings, in percent)

	TFP Shock z	**Investment Shock** ζ	**Preference Shock** γ	**Price MK Shock** η	**Wage MK Shock** υ	**Government Shock** G	**Money Shock** ς	**Financial Shock** ξ
GDP	4.1	4.1	1.1	24.9	12.9	0.8	5.9	46.4
Consumption	2.1	27.8	56.6	2.9	2.7	7.1	0.2	0.6
Investment	2.5	16.5	13.3	13.8	9.6	15.2	4.4	24.7
Hours	19.4	5.1	0.8	16	17.7	1.1	6.5	33.5
Wages	0.5	2.9	3.1	5.4	83.3	0.7	3.1	1
Interest Rate	3.6	61.9	4.1	3.4	8.1	9.7	4.5	4.7
Inflation	2.2	24	2	3.7	5.2	2.8	50.6	9.5
Debt Repurchases	6.9	5.8	0.5	51.3	15.3	5.8	0.9	13.5

Note: Average over 10000 draws from the posterior distribution of the reestimated model through Bayesian methods.

Table 3.7: Variance Decomposition (Original Pfeifer (2016) findings, in percent)

	TFP Shock z	**Investment Shock** ζ	**Preference Shock** γ	**Price MK Shock** η	**Wage MK Shock** υ	**Government Shock** G	**Money Shock** ς	**Financial Shock** ξ
GDP	5.99	26.11	8.06	25.36	11.63	5.96	10.36	6.53
Consumption	4.33	23.27	20.81	8.85	24.66	5.96	7.56	4.57
Investment	1.98	74.45	5.97	11.08	2.68	0.10	2.39	1.35
Hours	22.42	26.15	3.69	14.52	17.36	6.74	5.95	3.17
Wages	2.13	4.98	15.45	21.51	36.78	1.52	8.15	9.48
Interest Rate	1.24	53.3	31.64	4.16	5.68	1.53	1.09	1.36
Inflation	3.98	18.09	18.6	17.88	8.37	0.7	12.69	19.69
Debt Repurchases	4.05	38.51	5.3	16.8	7.77	0.83	2.44	24.32

Note: Average over 10000 draws from the posterior distribution of the reestimated model through Bayesian methods.

Table 3.8: Variance Decomposition (Updated calibration according to Pfeifer (2016), in percent)

	TFP Shock z	**Investment Shock** ζ	**Preference Shock** γ	**Price MK Shock** η	**Wage MK Shock** υ	**Government Shock** G	**Money Shock** ς	**Financial Shock** ξ
GDP	1.96	0.79	0.12	3.74	90.66	3.24	0.34	0.15
Consumption	1.46	1.82	0.04	1.55	93.43	3.02	0.07	0.04
Investment	1.4	10.96	0.25	5.02	71.51	3.49	0.45	0.2
Hours	0.26	1.39	0.11	2	95.28	3.09	0.25	0.09
Wages	2.36	19.89	29.73	5.2	38.98	61.87	6.17	0.1
Interest Rate	0.92	20.86	7.56	3.41	54.87	8.32	0.15	0.12
Inflation	0.87	25.15	41.53	0.3	25.44	82.19	7.64	0.02
Debt Repurchases	0.85	19.49	15.74	1.63	76.21	41.23	3.01	4.46

3.5.3 Impulse Response Functions

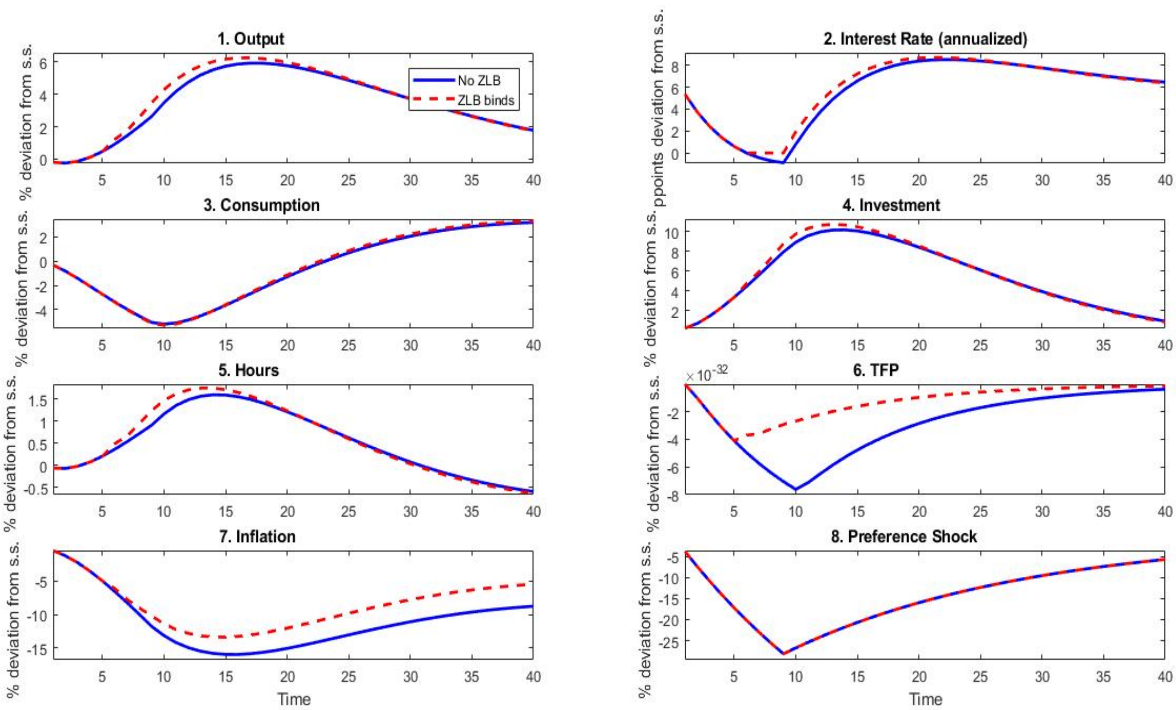


Figure 3.17: IRFs to a Preference Shock - $\tau = 0.2$

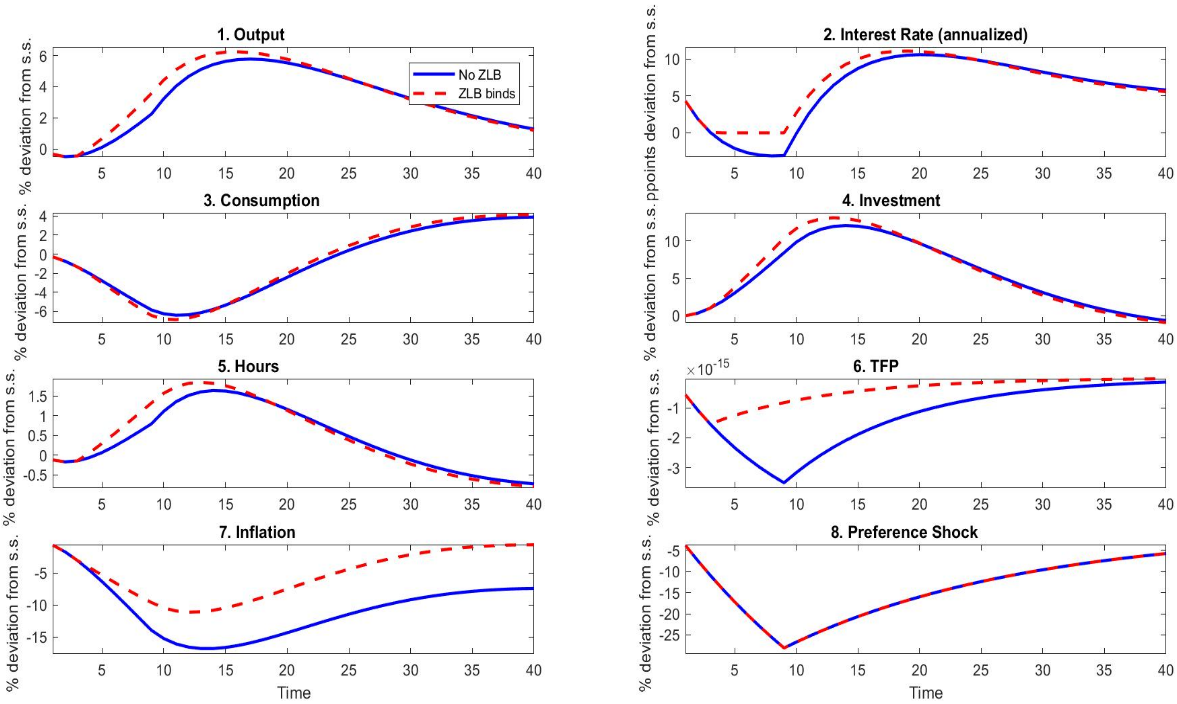


Figure 3.18: IRFs to a Preference Shock - $\tau = 0.5$

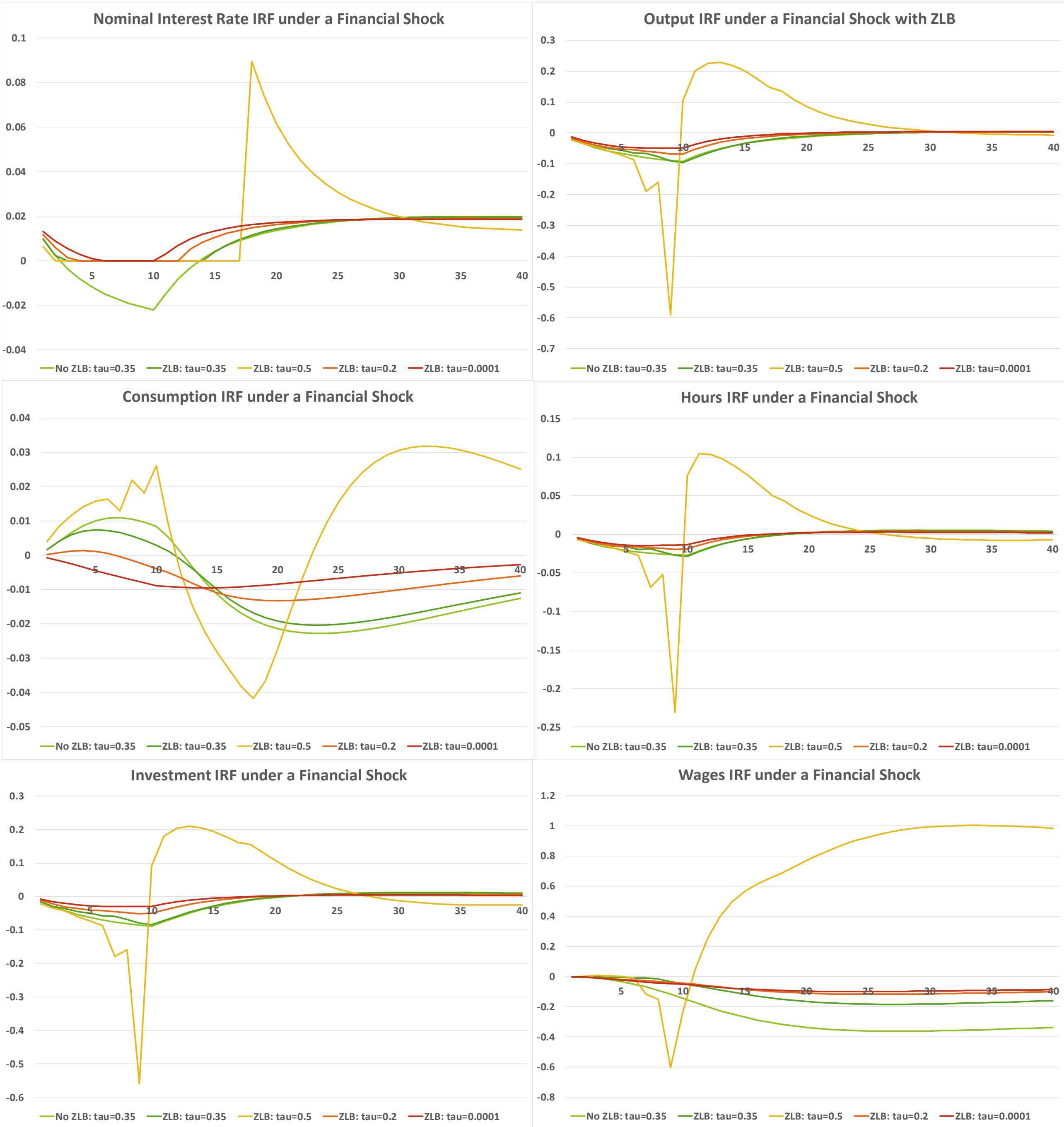


Figure 3.19: IRFs to a Financial Shock

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